National Graduate Physics Examination 2022
Day and Date of Examination: Sunday, 27 March 2022
Time: 10 AM to 1 PM
Solutions

1. Given that the field is $\vec{E} = 3x\,\hat{i} + 2y\,\hat{j} + z\,\hat{k}$ and $dS = idydz + jdzdx + kdxdy$ then
$\iint \vec{E}.\vec{ds} = (3xi + 2yj + zk).(idydz + jdzdx + kdxdy)$
$= 3\iint x\,dy\,dz + 2\iint y\,dz\,dx + \iint z\,dx\,dy$
Using appropriate limits, we get
$\iint \vec{E}.\vec{ds} = 3\times3\times(6-0) + 4\times(9-0) + (12-(-6))$
$= 54 + 36 + 18 = 108$ units **Ans: a**

2. Given that $\omega^2 = \omega_o^2 + c^2k^2$ .Knowing that the phase velocity is $\frac{\omega}{K}$ and the group velocity as $\frac{d\omega}{dK}$. We obtain $2\omega\frac{d\omega}{dK} = 0 + c^2.2K$
$\Rightarrow \frac{\omega}{K}\frac{d\omega}{dK} = c^2$ is the product of phase velocity and the group velocity. **Ans: c**

3. It must be noted that the displacement current is defined to account for consistent results. The conduction current in a CR current is $i = \frac{dq}{dt}$
where $q = CV\left(1 - e^{-\frac{t}{CR}}\right)$
$\Rightarrow i = \frac{dq}{dt} = CV\left[0 - \left(-\frac{1}{CR}\right)e^{-\frac{t}{CR}}\right] = \frac{V}{R}e^{-\frac{t}{CR}}$ at
$t = 500\times10^{-3}$ sec $i = \frac{6}{10\times10^3}e^{-\frac{0.5}{100\times10^{-6}\times10\times10^3}}$
$= \frac{0.6}{\sqrt{e}}m\,Amp$ $i = 0.364\times10^{-3}\,A = 364\,\mu Amp$
Ans: b

4. The forced oscillation is $x = ASin(pt - \theta)$ where the phase difference θ is expressed as
$\tan\theta = \frac{2bp}{\omega^2 - p^2}$ if $b = 0$ and $p > \omega$
$\Rightarrow \tan\theta = -\infty \Rightarrow \theta = \pi$ **Ans: d**

5. The mass of each side of the square is $\frac{M}{4}$ and its length is L so the moment of inertia of each side about an axis passing through its CG and perpendicular to length is $I = \left(\frac{M}{4}\right)\times\frac{L^2}{12}$
MI of the side about a parallel axis through CG is $I = I_0 + \frac{M}{4}\left(\frac{L}{2}\right)^2 = \frac{ML^2}{48} + \frac{ML^2}{16} = \frac{ML^2}{12}$
or $I = \frac{ML^2}{12}$. Thus the moment of Inertia of all the four sides about an axis through the CG and perpendicular to the plane of the given square is
$I = 4\times\frac{ML^2}{12} = \frac{ML^2}{3}$ Hence the angular momentum of the square is $J = I\omega$ or
$J = \frac{ML^2}{3}\omega = \frac{ML^2\omega}{3}$. **Ans: b**

6. In Michelson's Interferometer
$2(\mu - 1)t = m\lambda \Rightarrow t = \frac{180\times600\times10^{-9}}{2(1.54-1)}$
$\Rightarrow t = 100\,\mu m.$ **Ans: c**

7. $x = A\sin 2\omega t,\ y = A\cos 3\omega t$
So, 3 loops in x direction and 2 loops in y direction. **Ans: a**

8. Equation of continuity is $\frac{\partial\rho}{\partial t} + \nabla.J = 0$ using
$J = \sigma E \Rightarrow \frac{\partial\rho}{\partial t} = -\nabla.(\sigma E)$ or $\frac{\partial\rho}{\partial t} = -\sigma\nabla.E$
$\Rightarrow \frac{\partial\rho}{\partial t} = -\frac{\sigma\rho}{\varepsilon_o} \Rightarrow \int\frac{\partial\rho}{\rho} = -\frac{\sigma}{\varepsilon_o}\int dt$
$\Rightarrow \ln\rho\Big|_{\rho_o(r)}^{\rho(rt)} = -\frac{\sigma t}{\varepsilon_o}\Big|_o^t \Rightarrow \ln\rho(rt) - \ln\rho_o(r) = -\frac{\sigma t}{\varepsilon_o}$
$\Rightarrow \ln\frac{\rho(rt)}{\rho_o(r)} = -\frac{\sigma t}{\varepsilon_o}$ or $\rho(rt) = \rho_o(r)e^{-\frac{\sigma t}{\varepsilon_o}}$
Ans: c

9. The magnetic flux density $B = \frac{H}{3} + H^2$, T
$dB = \left(\frac{1}{3}dH + 2H\,dH\right)$ Hence, the energy density stored in the specimen is $U = \int H\,dB$
$= \int_0^{180} H\left(\frac{1}{3}dH + 2HdH\right) = \left\{\frac{1}{3}\times\frac{H^2}{2} + 2\frac{H^3}{3}\right|_0^{180}$
$= \frac{180^2}{6} + \frac{2\times180^3}{3} = 5400 + 3888000$
$= 38.93400\,J = 3.89MJ$ **Ans: b**

10. The first two are Gauss's Theorem in electrostatic and the last two in Gravitation so, all are correct. **Ans: a, b, c and d**

11. The Fermi energy of conduction electrons in a metal is given by $E_F = \frac{h^2}{2m}\left(\frac{3n}{8\pi V}\right)^{2/3}$ where $\frac{n}{V}$ denotes the free electron density

$$\therefore \frac{\text{Fermi Energy for Be}}{\text{Fermi Energy for Cs}} = \left[\frac{\left(\frac{n}{V}\right)_{Be}}{\left(\frac{n}{V}\right)_{Cs}}\right]^{2/3} = \left(\frac{24.2}{0.91}\right)^{2/3}$$

$$\Rightarrow \text{Fermi Energy for Cs} = \left(\frac{0.91}{24.2}\right)^{2/3} \times 14.44\,eV$$

= 1.62 eV **Ans: b**

12. $div(grad\phi) = [\nabla.(\nabla\phi)] = \nabla^2\phi$

$$= \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right).(3x^2 + y^4 + 6x^2z^2)$$

$$= (6 + 12z^2) + 12y^2 + 12x^2$$

$$= 6 + 12(x^2 + y^2 + z^2) = 6 + 12r^2$$ **Ans: c**

13. Claussius Mossotti equation is valid for gases and liquids. **Ans: a, b**

14. Electron and positron are both with negligible KE. Hence, $0.51 + 0.51 = 1.02\,MeV = 2h\nu$

$$\because 2\nu = \frac{1.02\times10^6\times1.6\times10^{-19}}{6.63\times10^{-34}} = 0.246\times10^{21}$$

$\Rightarrow \nu = 1.23\times10^{20}\,Hz.$ **Ans: b**

15. The dielectric constant $K = 1 + \frac{\chi}{\varepsilon_o}$

$$K = 1 + \frac{35.4\times10^{-12}}{8.85\times10^{-12}} \quad \because K = 1 + 4 = 5$$

Ans: d

16. The total heat content of a thermo dynamical system is often expressed as its enthalpy H = U + PV. **Ans: b**

17. The first three option a, b and c depict the properties of ultrasonic waves. **Ans: a, b, c**

18. All statements are correct for the working of the different kinds of the diodes. **Ans: a, b, c and d**

19. When a wire of radius r and length L is elongated by l with a force F, Young's modulus of its material is given by

$$Y = \frac{F}{\pi r^2}.\frac{L}{l}$$ Here

$F = Mg = 0.33\times 9.81\text{newton},$

$r = 1.6\times10^{-4}\,m,\ l = 10^{-3}\,m$

$$\therefore Y = \frac{0.33\times9.81\times L}{\pi\times(1.6\times10^{-4})^2\times10^{-3}}\,n/m^2 ----(1)$$

The torque applied to twist the wire by 1 radian defines torsional constant given by

$C = \frac{\pi\eta r^4}{2L} = 145\times10^{-7}\,n\times m$ Hence

$$\eta = \frac{145\times10^{-7}\times2L}{\pi\times(1.6\times10^{-4})^4} ---- (2)$$

Now Poisson's ratio $\sigma = \frac{Y}{2\eta} - 1$

$$\frac{0.330\times9.81\times L\times\pi\times(1.6\times10^{-4})^4}{\pi\times(1.6\times10^{-4})^2\times10^{-3}\times2\times145\times2L} - 1$$

= 1.429 – 1 = 0.429 **Ans: c**

20. ${}^5F_1 : 2S + 1 = 5 \Rightarrow 2S = 4$

$\Rightarrow S = 2[\because \text{Quintet M=5}]\ L = 3, J = 1$

$$g_j = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$$= 1 + \frac{1(2) + 2(3) - 3(4)}{2(1)(2)} = 1 + \left(-\frac{4}{4}\right) = 1 - 1$$

$\Rightarrow g_j = 0$

${}^4D_{\frac{1}{2}} : S = \frac{3}{2}[\text{Quartet } M = 4]\ L = 2, J = \frac{1}{2}$

$$g_j = 1 + \frac{\left(\frac{1}{2}\right)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)\left(\frac{5}{2}\right) - 2(3)}{2\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}$$

$$= 1 + \frac{\frac{9}{2} - 6}{\frac{3}{2}} = 1 + \frac{\left(-\frac{3}{2}\right)}{\frac{3}{2}} = 0$$

${}^6G_{\frac{3}{2}} : M = 6 = \text{multiplicity}[\text{sextet}]$

$2S + 1 = 6 \Rightarrow S = \frac{5}{2}\ \ L = 4, J = \frac{3}{2}$

$$g_j = 1 + \frac{\left(\frac{3}{2}\right)\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)\left(\frac{7}{2}\right) - 4(5)}{2\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}$$

$$= 1 + \frac{\left(\frac{25}{2} - 20\right)}{\frac{15}{2}} = 1 + \frac{\left(-\frac{15}{2}\right)}{\frac{15}{2}} = 0$$

${}^3P_0 : M = 3[\text{Triplet}]\ S = 1,\ L = 1,\ J = 0$

$g_j = 1 + \frac{0}{0} = 1 + (\text{indeterminate form})$

$g_j = \text{Indeterminate}$ **Ans: a, b, d**

21. By definition, $dS = \frac{dQ}{T} \Rightarrow dQ = T.dS$
= area of given ellipse on T-S diagram Now since $dU = 0$ (in a cyclic process), one can write $dQ = dU + dW = 0 + dW$. So, $dW = dQ = TdS$
= area of ellipse $= \pi ab$ so, $W = \frac{22}{7} \times 5 \times 100 \frac{J}{K}.K$
$\Rightarrow 1571.4\,J$. In 20 cycles work done will be
$W = 1571.4 \times 20 = 31428.5$ J/s=31.43 KW
So P =31.429 $KW = 31.5\,KW$. **Ans: c**

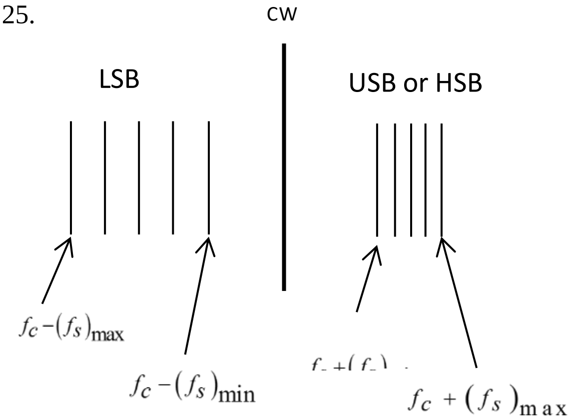
22. Option a, b and c is correct for a LASER.
Ans: a,b,c

23. To account for the behaviour of a paramagnetic substance in a magnetic field, the Langvin function is given by $L(\alpha) = \coth\alpha - \frac{1}{\alpha}$
Where $\alpha = \frac{\mu_o B}{3KJ}$. **Ans: c**

24. **Spin-Parity of Nuclei in ground state:**
Spin of Nucleus refers to its total angular momentum (Total J).
Parity refers to space part of wave function. Parity depends on L. If L is even then parity is positive and if L is odd then parity is negative.

Order of levels according to shell model is:
$^1s_{\frac{1}{2}}, {}^1p_{\frac{3}{2}}, {}^1p_{\frac{1}{2}}, {}^1d_{\frac{5}{2}}, {}^2s_{\frac{1}{2}}, {}^1d_{\frac{3}{2}}, {}^1f_{\frac{7}{2}}, {}^2p_{\frac{3}{2}}, {}^1f_{\frac{5}{2}},$
$^2p_{\frac{1}{2}}, {}^1g_{\frac{9}{2}}, {}^1g_{\frac{7}{2}}, {}^2d_{\frac{5}{2}}, {}^2d_{\frac{3}{2}}, {}^3s_{\frac{1}{2}}, {}^1h_{\frac{11}{2}}$ and so on
Ans: b

25.



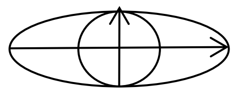
Modulation Factor or Depth of Modulation or Modulation Index
$= \frac{\text{Maximum value of modulating wave}}{\text{Maximum value of carrier wave}}$
$= \frac{20V}{30V} = \frac{2}{3} = 0.67$
$LSB = f_c - f_s = 12000KHz - 12KHz$
$LSB = 11988KHz$
$HSB = f_c + f_s = 12000KHz + 12KHz$
$HSB = 12012\,KHz$ **Ans: c**

Part B1

B1. The equation of a stationary wave is $y = A\sin\omega t\cos Kx$ denotes displacement.
The pressure variation is given by,
$p = -B\frac{\partial y}{\partial x} = BAK\sin\omega t\sin Kx$ denotes pressure obviously where y=0 (node) the pressure variation shall be maximum (antinode). The statement is defended.

B2. When the boy bends with the wall behind him, the centre of mass of his body comes out of his two feet hence he falls down whereas in the case when he is in the open space his hips goes a little backward as CG remains between two feet and he is able to balance himself.

B3. In doubly refracting negative crystals the spherical wave front due to O-rays lies within the ellipsoidal wave function due to E-ray. The two wave-fronts touching each other in the direction of optic axis as shown.



Optic axis

Obviously,
$v_e|_{along\ optic\ axis} < v_e|_{normal\ to\ optic\ axis}$
The statement is defended.

B4. In the fusion reaction in the sun four Hydrogen atom fuse together to give one Helium atom and two positron along with 26 MeV of energy.
$^1_1H + {}^1_1H + {}^1_1H + {}^1_1H \rightarrow {}^4_2He + 2{}_1\beta^o + 26MeV$.
Energy released by the fusion of 1kg of Hydrogen will be $= 2\frac{N_A}{A} \times \frac{1}{4} \times 26\,MeV$
$= \frac{N_A}{4} \times 26$. In a nuclear reactor energy released per fission of $^{235}_{92}Cl$ is 200 MeV per fission.
$_0n^1 + {}^{235}_{92}Cl \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}_0n^1 + 200MeV$
Energy released by the fission of 1kg of $^{235}_{92}Cl$ is

$= \frac{N_A}{A} \times 200 MeV = \frac{N_A}{235} \times 200$ Obviously,

$$\frac{\frac{N_A}{4} \times 26}{\frac{N_A}{235} \times 200} \cong 7.64 \cong 8$$ Statement is defended.

B5. The condition of diffraction for a grating is $(e+d)\sin\theta = n\lambda$ where $e+d =$ grating element

$$e+d = \frac{2.54}{2620} cm$$

$$\text{or} = \frac{2.54 \times 10^{-2}}{2620} \sin 90 = n 500 \times 10^{-9}$$

$$n = \frac{2.54}{2620} \frac{\sin 90}{5 \times 10^{-5}} = 0.019 \times 10^3 = 19.$$

Hence maximum orders of diffraction which can be seen are 19. Statement is refuted.

B6. When the gypsy car with canvas tops runs fast on a highway, the speed of air molecules just above the top is high enough as a result of which pressure decreases and the roof goes up bulging outwards.

B7. Of-course, there is no electric field inside a conductor yet it contains free electrons which are drifted when an external electric field is applied to create a potential difference between the ends of conductor and current flows.

B8. Knowing that the energy states of a three dimensional cubic box are

$$E_{n_1,n_2,n_3} = \frac{\pi^2 \hbar^2}{2m} \left[\frac{n_1^2}{a^2} + \frac{n_2^2}{b^2} + \frac{n_3^2}{c^2} \right]$$

using $a = b = c$, we get

$$E_{n_1,n_2,n_3} = \frac{\pi^2 \hbar^2}{2ma^2} \left[n_1^2 + n_2^2 + n_3^2 \right]$$

$$E_{n_1,n_2,n_3} = \frac{\pi^2}{2ma^2} \left(\frac{h}{2\pi} \right)^2 \left[n_1^2 + n_2^2 + n_3^2 \right]$$

$$E_{n_1,n_2,n_3} = \frac{\pi^2 h^2}{8ma^2 \pi^2} \left[n_1^2 + n_2^2 + n_3^2 \right]$$

Given that $\left[n_1^2 + n_2^2 + n_3^2 \right] = 66$

the possible sets may be

$$\begin{bmatrix} (5,5,4): 5^2 + 5^2 + 4^2 = 66 \\ (5,4,5): 5^2 + 4^2 + 5^2 = 66 \\ (4,5,5): 4^2 + 5^2 + 5^2 = 66 \end{bmatrix} \begin{bmatrix} (1,7,4): 1^2 + 7^2 + 4^2 = 66 \\ (7,4,1): 7^2 + 4^2 + 1^2 = 66 \\ (4,1,7): 4^2 + 1^2 + 7^2 = 66 \end{bmatrix}$$

$$\begin{bmatrix} (1,1,8): 1^2 + 1^2 + 8^2 = 66 \\ (1,8,1): 1^2 + 8^2 + 1^2 = 66 \\ (8,1,1): 8^2 + 1^2 + 1^2 = 66 \end{bmatrix}$$

There are 9 degenerate states and each state containing two electrons (taking spin into account). So, this energy level can accommodate a maximum of 18 electrons and not 19. So the given statement is refuted.

B9. Resolution means the smallest analog signal that can make a change in digital output. Resolution is also called magnitude of LSB. It is 4-bit ADC. So, write down total possible 16 combinations.

Digital Output D_3	D_2	D_1	D_0	Analog Input
0	0	0	0	$\rightarrow 0\, volt$
0	0	0	1	$\rightarrow 0\, to\, 0.5\, volt$
0	0	1	0	$\rightarrow 0.5\, to\, 1.0\, volt$
0	0	1	1	$\rightarrow 1.0\, to\, 1.5\, volt$
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

For every 0.5V, bit combination will change. So, for 6.6V, how many times bit combination will change $= \frac{6.6}{0.5} = 13.2$ which is greater than 13. So you must take digital output=14 and 14 in digital output is 1110. So, we will defend the statement.

B10. In order to achieve some increase in the overall values of circuit current gain and input impendence, two transistors are connected as shown in above figure, which is known as Darlington Configuration. The emitter of the first transistor is connected to the base of the second transistor. The collector terminals of both the transistors are connected together.

<u>Characteristics:</u>

(i) Extremely high input z.

(ii) Extremely high current gain.

(iii) Extremely low output z.

The statement is defended.

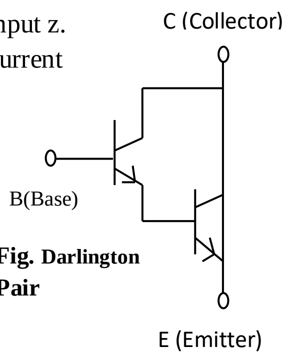


Fig. Darlington Pair

Part B2

P1. Knowing that

$\nabla\cdot(\phi\vec{A}) = \nabla\phi\cdot\vec{A} + \phi\cdot(\nabla\cdot\vec{A})$....(1)

Also $\nabla r^n = n r^{n-1}\hat{r}$

So $\nabla r^{-3} = -3r^{-4}\hat{r} = -\frac{3}{r^4}\hat{r}$ ......(2)

Further $\nabla^2 \cong \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}\right)$ Therefore

$$\nabla^2\left(\frac{1}{r^3}\right) = \frac{\partial}{\partial r^2}\left(\frac{1}{r^3}\right) + \frac{2}{r}\frac{\partial}{\partial r}\left(\frac{1}{r^3}\right)$$

$$= \frac{\partial}{\partial r}\left(-\frac{3}{r^4}\right) + \frac{2}{r}\left(-\frac{3}{r^4}\right)$$

$$= -3\left(-\frac{4}{r^5}\right) - \frac{6}{r^5} = \frac{12}{r^5} - \frac{6}{r^5}$$

$$\Rightarrow \nabla^2\left(\frac{1}{r^3}\right) = \frac{6}{r^5} \text{........(3)}$$

Also $\nabla r = 1r^{1-1}\,\hat{r} = \hat{r}$ ……..(4)

$$\nabla\cdot\left(r\nabla\left(\frac{1}{r^3}\right)\right) = \nabla r.\nabla\left(\frac{1}{r^3}\right) + r\nabla\cdot\nabla\left(\frac{1}{r^3}\right) \text{ or}$$

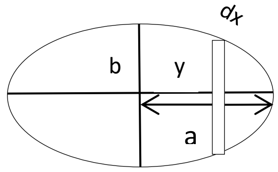
$$\nabla\cdot\left(r\nabla\left(\frac{1}{r^3}\right)\right) = \nabla r.\nabla\left(\frac{1}{r^3}\right) + r\nabla^2\left(\frac{1}{r^3}\right) \text{......(5)}$$

Using $eq.$ 2, 3, 4, 5 we get

$$\nabla\cdot\left(r\nabla\left(\frac{1}{r^3}\right)\right) = \hat{r}\cdot\left(-\frac{3}{r^4}\right)\hat{r} + r\cdot 6\left(\frac{1}{r^5}\right)$$

$$= \frac{3}{r^4}\,(\hat{r}.\hat{r}) + \frac{6}{r^4} = -\frac{3}{r^4} + \frac{6}{r^4} \Rightarrow \frac{3}{r^4}$$

P2. Moment of Inertia of an elliptical lamina of mass M, semi major and semi minor axes as a & b about its major axis. Let us consider a thin strip



Of width dx at a distance x from centre. If its length is $2y$ then the $area = 2ydx$

Let σ be the mass per unit area such that

$\sigma = \frac{M}{\Pi ab}$........(1) then mass of this element is $\sigma 2ydx$ and its moment of inertia about the major axis is $\frac{\sigma 2ydx\cdot(2y^2)}{12} = \frac{2}{3}\sigma y^3 dx$.

Further using $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ as the equation of the ellipse we get $\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$ or $y = b\sqrt{\left(1-\frac{x^2}{a^2}\right)}$

Substituting and integrating the moment of inertia of the elliptical lamina about the major axis is

$$I = \int_{-a}^{+a}\frac{2}{3}\sigma b^3\left(1-\frac{x^2}{a^2}\right)^{3/2} dx \text{ or}$$

$$I = \frac{2}{3}\sigma b^3\int_{-a}^{+a}\left(1-\frac{x^2}{a^2}\right)^{3/2} dx\text{..........(2)}$$

Now, using $x = a\sin\phi$ we get $dx = a\cos\phi d\phi$

$$\text{So, } I = \frac{2}{3}\sigma b^3\int_{-\pi/2}^{+\pi/2}\left(1-\frac{a^2\sin^2\phi}{a^2}\right)^{3/2} a\cos\phi d\phi$$

$$= \frac{2\times 2}{3}\sigma ab^3\int_{\pi/2}^{\pi/2}\cos^4\phi d\phi$$

$$I = \frac{4}{3}\sigma ab^3 \times \left[\frac{\Gamma\frac{4+1}{2}\ \Gamma\frac{0+1}{2}}{2\Gamma\frac{4+0+2}{2}}\right] = \frac{\frac{3}{2}\times\frac{1}{2}\sqrt{\pi}\sqrt{\pi}}{2\times 2\times 1}\times\frac{4}{3}\sigma ab^3$$

$$= \frac{\frac{3}{2}\times\frac{1}{2}\sqrt{\pi}\sqrt{\pi}}{2\times 2\times 1}\times\frac{4}{3}\sigma ab^3 \Rightarrow I_{major} = \frac{3}{16}\pi\times\frac{4}{3}\sigma ab^3$$

$= \frac{\sigma\pi ab}{4}b^2 = \frac{Mb^2}{4}$ Moment of Inertia about the minor axis is $I_{minor} = \frac{Ma^2}{4}$. Therefore, the moment of inertia about an axis passing through centre and perpendicular to the plane of lamina is

$I_O = \frac{M(a^2+b^2)}{4}$. The moment of inertia about a parallel axis passing through the focus will then be

$$I = I_O + M\left(ae^2\right) = \frac{M(a^2+b^2)}{4} + M\left(ae\right)^2$$

$= \frac{M}{4}\left[a^2 + b^2 + 4a^2e^2\right]$. If this elliptical lamina is used as a compound pendulum suspended at focus.

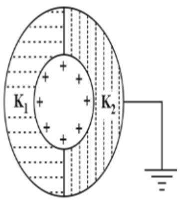
The time period shall be $T = 2\pi\sqrt{\frac{I}{Mgae}}$

$$= 2\pi\sqrt{\frac{\frac{M}{4}\left(a^2+b^2+4a^2e^2\right)}{Mgae}} \text{ or } T = 2\pi\sqrt{\frac{ae+\frac{a^2+b^2}{4ae}}{g}}$$

$$\Rightarrow \frac{k^2}{l} = \frac{a^2+b^2}{4ae}$$

This is the distance of centre of oscillation. Thus the centre of oscillation lies at a distance of $\frac{a^2+b^2}{4ae}$ from centre of mass on the major axis on the side remote from centre of suspension.

P3. Two metallic spherical shells of radii a and b respectively are kept concentric. The space between the two shells is identically filled with two different dielectrics of dielectric constant k_1 and k_2 such that each of them occupy equal space diametrically opposite to each other. The inner shell of radius a is given a +Q charge while the outer shell of radius b(b>a) is grounded.



At a radial distance r, let the electric field be E because the potential of the entire inner shell shall be V_1 say at every point and V_2 at every point on outer sphere such that $V = V_1 - V_2$. The displacement vector D in the dielectric is $\varepsilon_o K_1 E$ and in the second dielectric is $\varepsilon_o K_2 E$. Further

$$\iint D \cdot ds = \varepsilon_o K_1 E 2\pi r^2 + \varepsilon_o K_2 E 2\pi r^2 = Q$$

$$\Rightarrow E = \frac{Q}{2\pi\varepsilon_o (K_1+K_2) r^2}$$ Further,

$$V = -\int_a^b E \cdot dr \quad = \int_a^b \frac{Q}{2\pi\varepsilon_o (K_1+K_2) r^2} dr$$

$$V = -\frac{Q}{2\pi\varepsilon_o (K_1+K_2)} \int_b^a \frac{dr}{r^2}$$

$$V = -\frac{Q}{2\pi\varepsilon_o (K_1+K_2)} \left[-\frac{1}{r}\right]_b^a$$

$$V = \frac{Q}{2\pi\varepsilon_o (K_1+K_2)} \left[\frac{1}{b} - \frac{1}{a}\right]$$

$$Q = \frac{2\pi\varepsilon_o (K_1+K_2)}{\left(\frac{1}{a} - \frac{1}{b}\right)} V$$

$$C = \frac{2\pi\varepsilon_o (K_1+K_2)}{\left(\frac{1}{a} - \frac{1}{b}\right)} V$$

P4. (a)

$$|x| = x; x \ge 0$$
$$= -x; x < 0$$

$$P = \frac{\int_{-a}^{a} |\Psi|^2 dx}{\int_{-\infty}^{\infty} \Psi * \Psi dx} = \frac{\int_{-a}^{0} e^{\frac{2x}{a}} dx + \int_{0}^{a} e^{-\frac{2x}{a}} dx}{\int_{-\infty}^{0} e^{\frac{2x}{a}} dx + \int_{0}^{\infty} e^{-\frac{2x}{a}} dx}$$

$$= \frac{\left.\frac{e^{\frac{2x}{a}}}{\left(\frac{2}{a}\right)}\right|_{-a}^{0} + \left.\frac{e^{-\frac{2x}{a}}}{\left(-\frac{2}{a}\right)}\right|_{0}^{a}}{\left.\frac{e^{\frac{2x}{a}}}{\left(\frac{2}{a}\right)}\right|_{-\infty}^{0} + \left.\frac{e^{-\frac{2x}{a}}}{\left(-\frac{2}{a}\right)}\right|_{0}^{\infty}}$$

$$= \frac{\frac{a}{2}\left[1 - e^{\frac{2}{a}(-a)}\right] - \frac{a}{2}\left[e^{\frac{-2a}{a}} - e^0\right]}{\frac{a}{2}\left[1 - e^{-\infty}\right] - \frac{a}{2}\left[e^{-\infty} - 1\right]}$$

$$= \frac{\frac{a}{2}\left[1 - e^{-2}\right] - \frac{a}{2}\left[e^{-2} - 1\right]}{\left(\frac{a}{2} + \frac{a}{2}\right)} = \frac{a - \frac{a}{2e^2} - \frac{a}{2e^2}}{a}$$

$$= \frac{a - \frac{2a}{2e^2}}{a} = \left(1 - \frac{1}{e^2}\right) = \frac{e^2 - 1}{e^2} = 0.86$$

(b) $\psi(x) = A \sin^3\left(\frac{\pi x}{a}\right)$, $\text{Sin}\, 3\theta = (3Sin\theta - 4Sin^3\theta)$

$$\psi(x) = \frac{3}{4} A \sin\left(\frac{\pi x}{a}\right) - \frac{A}{4} \sin\left(\frac{3\pi x}{a}\right)$$

$$= \frac{3A}{4}\sqrt{\frac{a}{2}}\left[\sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)\right] - \frac{A}{4}\sqrt{\frac{a}{2}}\left[\sqrt{\frac{2}{a}} \sin\left(\frac{3\pi x}{a}\right)\right]$$

$$= \frac{3A}{4}\sqrt{\frac{a}{2}}\, \psi_1(x) - \frac{A}{4}\sqrt{\frac{a}{2}}\, \psi_3(x)$$

For $\psi(x)$ to be normalized,

$$\left(\frac{3A}{4}\sqrt{\frac{a}{2}}\right)^2 + \left(\frac{A}{4}\sqrt{\frac{a}{2}}\right)^2 = 1$$

$$\Rightarrow \frac{9A^2}{16} + \frac{A^2}{16} = \frac{2}{a} \Rightarrow 10A^2 = \frac{32}{a}$$

$$\Rightarrow 5A^2 = \frac{16}{a} \Rightarrow A^2 = \frac{16}{5a} \Rightarrow |A| = \frac{4}{a\sqrt{5}}$$

$\because A = \pm\frac{4}{a\sqrt{5}}$, leaving negative sign $A = +\frac{4}{a\sqrt{5}}$

$$\because \psi(x) = \sum_n C_n \psi_n(x)$$

$$\psi(x) = C_1\psi_1(x) + C_2\psi_2(x) + C_3\psi_3(x)$$

$$= \frac{3A}{4}\sqrt{\frac{a}{2}}\ \psi_1(x) + 0 + \left(-\frac{A}{4}\right)\sqrt{\frac{a}{2}}\ \psi_3(x)$$

$$= \left(\frac{3}{4}\right)\left(\frac{4}{\sqrt{5a}}\right)\sqrt{\frac{a}{2}}\psi_1(x) - \left(\frac{1}{4}\right)\left(\frac{4}{\sqrt{5a}}\right)\sqrt{\frac{a}{2}}\psi_3(x)$$

$$= \frac{3}{\sqrt{10}}\psi_1(x) - \frac{1}{\sqrt{10}}\psi_3(x)$$

$$< E > = P_1E_1 + P_3E_3$$

$$= \frac{9}{10}\left(\frac{\pi^2\hbar^2}{2ma^2}\right) + \frac{1}{10}\left(\frac{9\pi^2\hbar^2}{2ma^2}\right) = \left(\frac{18\pi^2\hbar^2}{20ma^2}\right)$$

$$< E > = \left(\frac{9\pi^2\hbar^2}{10ma^2}\right)$$

P5. (a)

(i) One valence e^- in weak magnetic field:

$$g_j = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} \quad .....(1)$$

(ii) $$g_j = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad .....(2)$$

Eq. (2) is exactly the same form as Eq. (1). For a single e^- except that l is here replaced by L, s by S and j by J.

(iii) Lande's g factor in case of j j coupling:

$$g_j = g_1\frac{J(J+1) + j_1(j_1+1) - j_2(j_2+1)}{2J(J+1)}$$

$$+ g_2\frac{J(J+1) + j_2(j_2+1) - j_1(j_1+1)}{2J(J+1)}$$

where g_1 and g_2 are Lande's g factors for individual electrons.

LS and jj Coupling and the g SUM RULE:

g factors are in many instances different for jj coupling than they are for LS Coupling, the Zeeman patterns will also be different.

(b) **Pauli g Sum Rule:**

states that out of all the states arising from a given electron configuration, the SUM of the g-factors for levels with the SAME J value is a constant independent of the coupling scheme.

p s configuration:

$$l_1 = 1, s_1 = \frac{1}{2} \Rightarrow j_1 = \frac{3}{2}, \frac{1}{2}$$

$$l_2 = 0, s_2 = \frac{1}{2} \Rightarrow j_2 = \frac{1}{2}$$

$$1^{st}\ set \Rightarrow j_1 = \frac{1}{2},\ j_2 = \frac{1}{2} \Rightarrow J = 0,1$$

$$2^{st}\ set \Rightarrow j_1 = \frac{3}{2},\ j_2 = \frac{1}{2} \Rightarrow J = 1,2$$

So, the states are $\left(\frac{1}{2},\frac{1}{2}\right)_0$; $\left(\frac{1}{2},\frac{1}{2}\right)_1$; $\left(\frac{3}{2},\frac{1}{2}\right)_1$; $\left(\frac{3}{2},\frac{1}{2}\right)_2$ Calculation of g factors illustrating g SUM rule for p s electron configuration (jj coupling scheme):

Term: $j_1 = \frac{1}{2},\ j_2 = \frac{1}{2},\ j = 0$

$$g_j = g_1\left[\frac{0(1) + \frac{1}{2}\left(\frac{3}{2}\right) - \frac{1}{2}\left(\frac{3}{2}\right)}{2(0)(1)}\right] +$$

$$g_1\left[\frac{0(1) + \frac{1}{2}\left(\frac{3}{2}\right) - \frac{1}{2}\left(\frac{3}{2}\right)}{2(0)(1)}\right]$$

$$g_j = g_1\left(\frac{0}{0}\right) + g_2\left(\frac{0}{0}\right) = \left(\frac{0}{0}\right)$$ =Indeterminate form. Indeterminacy is removed by taking limit. But here there is no limit. Purpose of limit is only to remove indeterminacy. So, here Indeterminacy can't be removed.

Term: $j_1 = \frac{1}{2};\ j_2 = \frac{1}{2};\ J = 1$

$$g_j = g_1\frac{J(J+1) + j_1(j_1+1) - j_2(j_2+1)}{2J(J+1)}$$

$$+ g_2\frac{J(J+1) + j_2(j_2+1) - j_1(j_1+1)}{2J(J+1)}$$

$$g_1 = 1 + \frac{j_1(j_1+1) + s_1(s_1+1) - l_1(l_1+1)}{2j_1(j_1+1)}$$

$$= 1 + \frac{\frac{1}{2}\left(\frac{3}{2}\right) + \frac{1}{2}\left(\frac{3}{2}\right) - 1(2)}{2\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)} \Rightarrow g_1 = 1 + \frac{\frac{3}{2} - 2}{\frac{3}{2}} = \frac{2}{3}$$

$$g_2 = 1 + \frac{j_2(j_2+1) + s_2(s_2+1) - l_2(l_2+1)}{2j_2(j_2+1)}$$

$$= 1 + \frac{\frac{1}{2}\left(\frac{3}{2}\right) + \frac{1}{2}\left(\frac{3}{2}\right) - 0(1)}{2\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)} = 2$$

$$g_j = \left(\frac{2}{3}\right)\left[\frac{1(2) + \frac{1}{2}\left(\frac{3}{2}\right) - \frac{1}{2}\left(\frac{3}{2}\right)}{2(1)(2)}\right] +$$

$$2\left[\frac{(1)(2)+\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{2(1)(2)}\right]$$

$$=\left(\frac{2}{3}\right)\left(\frac{2}{4}\right)+2\left(\frac{2}{4}\right) \quad =\frac{1}{3}+1=\frac{4}{3}$$

Term: $j_1=\frac{3}{2}$, $j_2=\frac{1}{2}$, $J=1$

$$g_1=1+\frac{\frac{3}{2}\left(\frac{5}{2}\right)+\frac{1}{2}\left(\frac{3}{2}\right)-1(2)}{2\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}=\frac{4}{3}$$

$$g_2=1+\frac{\frac{1}{2}\left(\frac{3}{2}\right)+\frac{1}{2}\left(\frac{3}{2}\right)-0(1)}{2\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}=2$$

$$g_j=\frac{4}{3}\left[\frac{1(2)+\frac{3}{2}\left(\frac{5}{2}\right)-\frac{1}{2}\left(\frac{3}{2}\right)}{2(1)(2)}\right]+2\left[\frac{1(2)+\frac{1}{2}\left(\frac{3}{2}\right)-\frac{3}{2}\left(\frac{5}{2}\right)}{2(1)(2)}\right]$$

$$=\frac{4}{3}\left[\frac{5}{4}\right]+2\left[-\frac{1}{4}\right] \quad =\left(\frac{5}{3}-\frac{1}{2}\right)=\frac{7}{6}$$

Term: $j_1=\frac{3}{2}$, $j_2=\frac{1}{2}$, $J=2$

$$g_1=1+\frac{\frac{3}{2}\left(\frac{5}{2}\right)+\frac{1}{2}\left(\frac{3}{2}\right)-1(2)}{2\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}=\frac{4}{3}$$

$$g_2=1+\frac{\frac{1}{2}\left(\frac{3}{2}\right)+\frac{1}{2}\left(\frac{3}{2}\right)-0(1)}{2\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}=2$$

$$g_j=\left(\frac{4}{3}\right)\left[\frac{2(1)+\frac{3}{2}\left(\frac{5}{2}\right)-\frac{1}{2}\left(\frac{3}{2}\right)}{2(2)(3)}\right]+$$

$$2\left[\frac{2(1)+\frac{3}{4}-\frac{15}{4}}{2(2)(3)}\right]=\left(\frac{4}{3}\right)\left(\frac{9}{12}\right)+\left(\frac{6}{12}\right) \quad =\frac{3}{2}$$

Terms	**J=0**	**J=1**	**J=2**	**Coupl-ing scheme (jj)**
$j_1=\frac{1}{2}, j_2=\frac{1}{2}$	$g_j=\left(\frac{0}{0}\right)$ Indeterminacy	$g_j=\left(\frac{4}{3}\right)$	….	
$j_1=\frac{3}{2}, j_2=\frac{1}{2}$		$g_j=\left(\frac{7}{6}\right)$	$\frac{3}{2}$	
Σg_j	$g_j=\left(\frac{0}{0}\right)$ Indeterminacy	$\frac{4}{3}+\frac{7}{6}=\frac{5}{2}$	$\frac{3}{2}$	

p s configuration: Coupling scheme(LS)

$$l_1=1, l_2=0 \Rightarrow L=1; s_1=\frac{1}{2}, s_2=\frac{1}{2} \Rightarrow S=0,1$$

Terms	**J=0**	**J=1**	**J=2**	**Coupling scheme**
$^1p_1$ $^3p_0, ^3p_1, ^3p_2$	…… …. $\left(\frac{0}{0}\right)$	$g_j=1,$ $g_j=\left(\frac{3}{2}\right)$	…… $g_j=\left(\frac{3}{2}\right)$	L S
$j_1=\frac{1}{2}, j_2=\frac{1}{2}$ $j_1=\frac{3}{2}, j_2=\frac{1}{2}$	$\left(\frac{0}{0}\right)$	$g_j=\left(\frac{4}{3}\right)$ $g_j=\left(\frac{7}{6}\right)$	….. $\left(\frac{3}{2}\right)$	J J
Σg_j	$\left(\frac{0}{0}\right)$	$\left(\frac{5}{2}\right)$	$\left(\frac{3}{2}\right)$	

$S=0 \Rightarrow M=multiplicity=2S+1=1;$

$S=1 \Rightarrow M=2(1)+1=3$

$L=1, S=0 \Rightarrow J=1$ so $^1p_1$ and

$L=1, S=0 \Rightarrow J=0,1,2$ so $^3p_2, ^3p_1, ^3p_0$

Term: $^1p_1$ means J=1, L=1, S=0

$$g_j=1+\left[\frac{1(2)+0(1)-1(2)}{2(1)(2)}\right]=1$$

$^3p_0$: S=1, L=1, J=0

$$g_j=1+\left[\frac{0(0+1)+1(2)-1(2)}{2(0)(0+1)}\right]=1+\frac{0}{0}$$

= One + indeterminacy = Indeterminacy

Terms	J=0	J=1	J=2	Coupling scheme (L S)
$^{1}p_{1}$	....	$g_j = 1$	.....	
$^{3}p_{0}$	$g_j = \left(\frac{0}{0}\right)$ Indeterminacy	....	.....	
$^{3}p_{1}$	....	$g_j = \left(\frac{3}{2}\right)$	......	
$^{3}p_{2}$	....	....	$\frac{3}{2}$	
Σg_j	$g_j = \left(\frac{0}{0}\right)$ Indeterminacy	$g_j = \left(1+\frac{3}{2}\right)$ $= \frac{5}{2}$	$g_j = \left(\frac{3}{2}\right)$	

$^{3}p_{1}$ means J=1, L=1, S=1

$$g_j = 1+\left[\frac{1(2)+1(2)-1(2)}{2(1)(2)}\right]=\frac{3}{2}$$

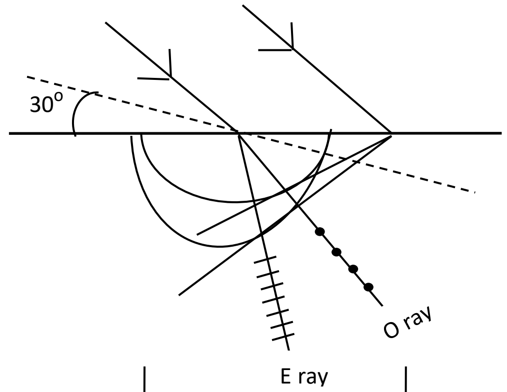
$^{3}p_{2}$ means S=1, L=1, J=2

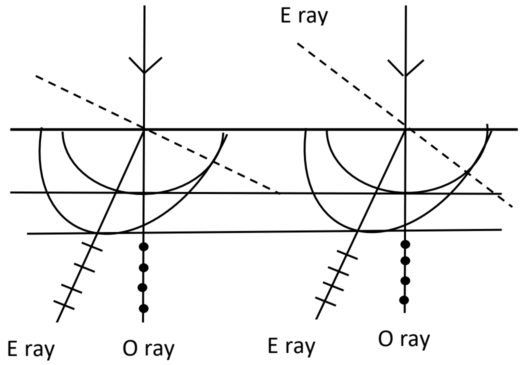
$$g_j = 1+\left[\frac{2(3)+1(2)-1(2)}{2(2)(3)}\right]=\frac{3}{2}$$

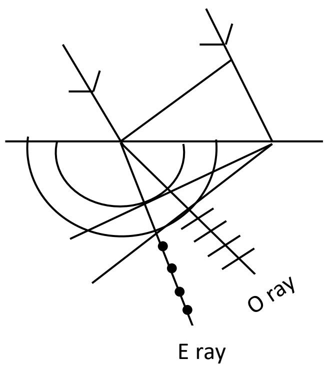
Observed g factors illustrating Pauli's g Sum Rule

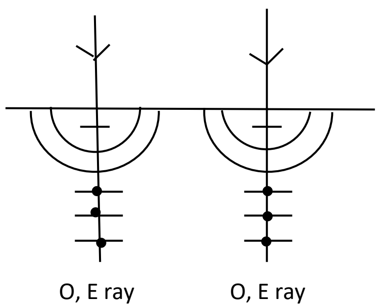
Eleme-nts	Terms	J=0	J=1	J=2	Coupling scheme
Zn	$^{1}p_{1}$ $^{3}p_{0}, {}^{3}p_{1}, {}^{3}p_{2}$	.... $\left(\frac{0}{0}\right)$	$g_j = 1\cdot 000,$ $g_j = 1.500$	1.500	L S
Pb	$j_1 = \frac{1}{2}, j_2 = \frac{1}{2}$ $j_1 = \frac{3}{2}, j_2 = \frac{1}{2}$	$\left(\frac{0}{0}\right)$	1.150 1.350	1.500	J J
Σg_j		$\left(\frac{0}{0}\right)$	$\left(\frac{5}{2}\right)$	$\left(\frac{3}{2}\right)$	

P6. (a) When an ordinary beam of light enters a doubly reflecting crystal the light splits into o- ray and E- ray. Crystal in which E-ray travels faster than the o ray $(v_E > v_o \text{ or } \mu_e < \mu_o)$ are known as negative crystals. While we have $v_E < v_o \text{ or } \mu_e > \mu_o$ for positive crystals. In negative crystal the spherical wave front of the o ray lies within the ellipsoid of revolution due to E ray. In order to draw the respective wave fronts one may look to any standard text such as Principles of Optics by B K Mathur.

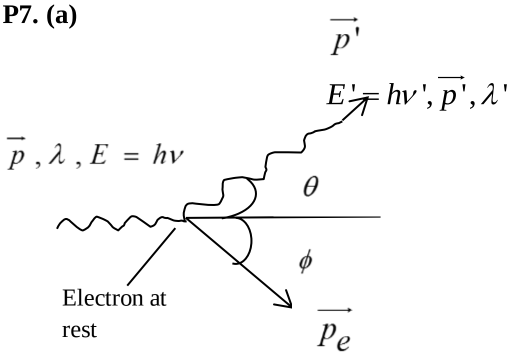








P7. (a)



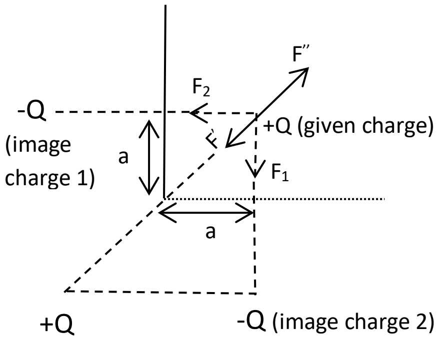
$$\nu' = \frac{\nu}{1 + \frac{h\nu}{m_o c^2}(1 - \cos\theta)}$$

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_o c^2}(1 - \cos\theta)}$$

$$h\nu' = \frac{50keV}{1 + \frac{50keV}{500keV}\left(1 - \cos 30^o\right)}$$

$$= \frac{50keV}{1 + \frac{1}{10}(0.134)} = \frac{50keV}{1.0134} = 49.338 \text{ keV}$$

(b) The +Q charge on the diagonal is image charge of – Q which is the image charge1 or image charge 2.



Here, $F_1 = \frac{kQ^2}{4a^2}$, $F_2 = \frac{kQ^2}{4a^2}$,

$$F'' = \frac{kQ^2}{(2\sqrt{2}a)^2} = \frac{kQ^2}{8a^2} = \frac{0.125Q^2}{4\pi\varepsilon_o a^2}$$

Therefore, $\vec{F}' = \vec{F}1 + \vec{F}2$

$$F' = 2F_1 \cos\left(\frac{90}{2}\right)^o = 2\left(\frac{kQ^2}{4a^2}\right)\frac{1}{\sqrt{2}}$$

$$= \sqrt{2}\frac{kQ^2}{4a^2} \quad F' = 0.353\frac{kQ^2}{a^2} = \frac{0.353}{4\pi\varepsilon_o}\frac{Q^2}{a^2}$$

$$\vec{F}net = \vec{F}' - \vec{F}'' \Rightarrow \frac{Q^2}{44\pi\varepsilon_o a^2}[0.353\text{–}0.125]$$

$$F_{net} = \frac{Q^2}{4\pi\varepsilon_o a^2}(0.228)$$

P8. (a) Clausius Clapeyron Equation is

$\frac{dP}{dT} = \frac{JL}{T(V_2 - V_1)}$. In the present problem

$dP = (2-1)atom = 1\times1.013\times10^5\, Pa$, $dT = ?$

$L = 540\times1000\times4.2\, Joule$

$V_2 - V_1 = 1.671 - 0.001 = 1.670 m^3 kg^{-1}$ and

$T = 373K$

$$\therefore \frac{1\times1.013\times10^5}{dT} = \frac{4.2\times540\times1000}{373(1.670)} \Rightarrow dT = 27.82K$$

Hence, the water inside the pressure cooker boils at $100 + 27.82 = 127.82\,^oC$

(b) According to Stefan's Law, $E = \sigma AT^4$ or

$T = \left(\frac{E}{\sigma A}\right)^{\frac{1}{4}}$ Further Wien's Displacement law is

$$\lambda_m T = b \text{ or } \lambda_m = \frac{b}{T} \text{ or } \lambda_m = 0.288\times10^{-2}\left(\frac{\sigma A}{E}\right)^{\frac{1}{4}}$$

$$= 0.288\times10^{-2}\left(\frac{6.67\times10^{-2}\times6.25\times10^{-6}}{100}\right)^{\frac{1}{4}}$$

$$= 0.288\times10^{-6}(6.67\times6.25)^{\frac{1}{4}}$$

$$= 0.7318\times10^{-6} = 7318\times10^{-10} m$$

$= 7318$ Angstrom.

P9. The adiabatic process can be mathematically expressed in P & T, as

$P^{1-\gamma}T^{\gamma} = constant = c\ (say)$; is equivalent to the form PV^{γ} = constant.....(1) Assuming air as the ideal gas and taking logarithmic derivative

$(1-\gamma)\log p + \gamma\log T = \log c$ or

$(1-\gamma)\frac{dp}{p} + \gamma\frac{dT}{T} = 0$......(2)

As, we go up, the pressure decreases and the change in pressure is $dp = -\rho gdh$ and $p = nRT$,

where $n = \frac{\text{No. of moles}}{\text{Volume}}$

Substituting these in equation (2), we have

$(1-\gamma)\left(\frac{-\rho gdh}{nRT}\right) + \gamma\frac{dT}{T} = 0$ or

$\frac{dT}{dh} = -\left(1 - \frac{1}{\gamma}\right).\frac{\rho g}{nR} = -\left(1 - \frac{1}{\gamma}\right).\frac{Mg}{R}$.....(3) where M is molar mass. This is the required expression for the adiabatic lapse rate.

Taking $\gamma = 1.4$ $g = 9.8m/\sec^2$;

$R = 8.31 J\, mole^{-1} K^{-1}$ $M = 0.029 kg\, mole^{-1}$ for air

$$\frac{dT}{dh} = \left[-\left(1 - \frac{1}{1.4}\right)\frac{0.029\times9.8}{8.31}\right]{}^{\circ}C\,meter^{-1} = -10^{-2}\,{}^{\circ}C\,meter^{-1}$$

$= -\frac{1}{100} m^{-1} = -\frac{10}{1000} m^{-1} = -\frac{10}{1km} \; 10\,^{\circ}C\, per\, km$

i.e.., The temperature drop in 1 km is about $10^{o}C$.
Note: A somewhat higher than the observed $5^{\circ}/6^{\circ}C$ per km.

P10. The depletion width in FET is expressed as

$$dx = \sqrt{\frac{2E}{qK}\left|(V_{b1}+V_{GS})\right|} \text{.....(1)} \quad \text{where}$$

E $\rightarrow$ field across junction,
q $\rightarrow$ charge,
K $\rightarrow$ Doping Constant,
V_{b1} $\rightarrow$ Built-in voltage
V_{GS} $\rightarrow$ Voltage across gate and source.

The depletion layer will grow in reverse bias configuration as $V_{b1} = -1\, volt$ (Given)

Case1: When $V_{GS} = 0$, the depletion width is expressed as $dx_1 = 1\mu m$

$$dx_1 = \sqrt{\frac{2E}{qK}\left|(-1+0)\right|} = \sqrt{\frac{2E}{qK}} \text{........(2)}$$

The channel resistance is given as $R_0 = 600\Omega$

i.e.., $R_0 = \frac{\rho L}{A} = \frac{\rho L}{W(t_{ch})} = \frac{\rho L}{W \times 10\mu m}$ ......(3)

The resistance at the grow of the depletion region $dx_1 = 1\mu m$. Grow is at both sides of gate. $R_1 = \frac{\rho L}{W\left(10\mu m - 2 \times 1\mu m\right)} = \frac{\rho L}{\mathrm{W} \times 0.8\mu m}$ ......(4)

From equations (3) & (4), we have

$$\frac{R_1}{R_0} = \left(\frac{\rho L}{\mathrm{W} \times 0.8\mu m}\right) \times \left(\frac{\mathrm{W} \times 10\mu m}{\rho L}\right) = \frac{10}{8}$$

$$R_1 = \frac{10}{8} \times R_0 = \frac{10}{8} \times 600 = 750\Omega$$

Case2: When $V_{GS} = -3V$ i.e.., increase in reverse bias. The depletion width is expressed as

$$dx_2 = \sqrt{\frac{2E}{qK}\left|(-1-3)\right|} = \sqrt{\frac{2E}{qK}(4)} = 2\sqrt{\frac{2E}{qK}}$$

$dx_2 = 2 \times 1\mu m$ ……(5)

Since growth is from both the sides i.e... $2 \times 2\mu m = 4\mu m.$

The channel resistance is

$$R_2 = \frac{\rho L}{W\left(10\mu m - 2 \times 2\mu m\right)} = \frac{\rho L}{\mathrm{W} \times 6\mu m} \text{......(6)}$$

From equation (3) & (6), we have

$$\frac{R_2}{R_0} = \left(\frac{\rho L}{\mathrm{W} \times 6\mu m}\right) \times \left(\frac{\mathrm{W} \times 10\mu m}{\rho L}\right) = \frac{10}{6}$$

$$R_2 = \frac{10}{6} \times R_0 = \frac{10}{6} \times 600 = 1000\Omega.$$

Ans. Resistance $R_1 = 750\Omega\; at V_{GS} = 0$

$R_2 = 1000\Omega\; at V_{GS} = -3V$