INDIAN ASSOCIATION OF PHYSICS TEACHERS

National Graduate Physics Examination 2021

Day and Date of Examination: Sunday, January24, 2021

Time: 10 AM to 1 PM

Solutions of part A

1. The electric field is $\vec{E}(r) = \alpha \vec{r}$ (given)

Thereby

$$\vec{\nabla}.\vec{E} = \alpha\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).\left(\hat{i}x + \hat{j}y + \hat{k}z\right) = 3\alpha$$

Now using Gauss's law $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \rho = 3\alpha\varepsilon_0$

Ans: **c**

2. The gravitational potential energy of mass m at a height 'h' above earth surface is $mgh\frac{R}{R+h}$. On the earth surface

$$where\ h \to 0\ \ PE = mg\frac{R}{\frac{R}{h}+1} = 0$$

at height $h = R$ $the\ PE = mgR\frac{R}{R+R} = \frac{mgR}{2}$

Hence change in PE on falling from height 'h' to earth surface is $= \left(\frac{mgR}{2} - 0\right) = \frac{mgR}{2}$

During the free fall, the gain in

$KE = loss in PE$ So $\frac{1}{2}mv^2 = \frac{mgR}{2} \Rightarrow v = \sqrt{gR}$

Ans: **c**

3.Boyle temperature of a Vander Waal gas is the temperature at which the gas obeys ideal gas equation. Vander Waal equation for one mole of a real gas is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT\ \ or\ PV - Pb + \frac{a}{V} - \frac{ab}{V^2} = RT$$

Using $PV = RT$ (ideal gas equation) and neglecting small term involving $a \times b$, we get $Pb = \frac{a}{V}$ or $PV = \frac{a}{b}$ there by

$RT_B = \frac{a}{b}$ Hence the Boyle temperature

$T_B = \frac{a}{Rb}$ The temperature of inversion is the temperature at which Joule-Thomson Coefficient $\left(\frac{\partial T}{\partial P}\right)_H$ vanishes. We know that

$$\left(\frac{\partial T}{\partial P}\right)_H = -\frac{1}{C_P}\left(\frac{2a}{RT} - b\right)$$

At the temperature of inversion

$$\frac{2a}{RT_i} - b = 0 \Rightarrow T_i = \frac{2a}{Rb}$$

The critical temperature of a Vander Waal gas is the temperature above which the gas cannot be liquefied how high the pressure may be. The critical temperature is given by

$T_c = \frac{8a}{27Rb}$ Vander Waal equation for n moles is $\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$

Ans: **a, b & c**

4. $\because e^{-\frac{\pi}{2}} = \left(e^{i\frac{\pi}{2}}\right)^i = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^i = (i)^i$

further the value of $e^{-\frac{\pi}{2}} = e^{-1.57} = 0.20$ which is real and non-zero.

Ans:**a, d**

5. Bending Moment of a loaded cantilever is $\frac{Y I_g}{R} = Mg(\ell - x)$ using $\frac{1}{R} = \frac{d^2y}{dx^2}$ and solving the depression is expressed as

$$\delta = \frac{Mg\ell^3}{3YI_g}$$

Further at any moment if the depression of free end is y in addition to δ then

$Mg - F = \frac{3YI_g}{\ell^3}(\delta + y)$ Or $F = -\frac{3YI_g}{\ell^3}y$

which may give that

$\frac{d^2y}{dt^2} = -\frac{3YI_g}{M\ell^3}y$ This is SHM with

$T = \frac{2\pi}{\sqrt{\frac{3YI_g}{M\ell^3}}} = 2\pi\sqrt{\frac{M\ell^3}{3YI_g}}$ using now $I_g = \frac{bd^3}{12}$

$$T = 2\pi\sqrt{\frac{4M\ell^3}{Ybd^3}} = 2\pi\sqrt{\frac{4\times2\times1^3}{2\times10^{11}\times3\times10^{-2}\times\left(2\times10^{-4}\right)^3}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{10^3}{2\times3}} = 2\pi\sqrt{\frac{500}{3}} = 81\sec$$

Ans: **a**

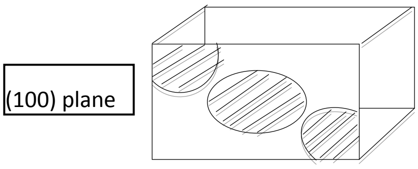
6. If the reading of the galvanometer does not change on closing S, the bridge is balanced then no currant flows through switch S therefore $I_G = I_2$ also $I_G = I_2 + I_s$

Ans: **b, d**

7. For fcc $a = \frac{4r}{\sqrt{2}}$ or

$$a = \frac{4 \times 0.12 nm}{\sqrt{2}} = 0.33946\, nm$$

$$= 0.33946 \times 10^{-6}\, mm$$



The area of (100) plane $= a^2$ having a total of 2 atoms

In area a^2 there are 2 atoms

$\therefore$ In 1 sq mm, there are $\frac{2}{a^2}$ atoms

$$= \frac{2}{\left(0.33946 \times 10^{-6}\right)^2} = 17.356 \times 10^{12} \text{ atom}$$

Ans: **c**

8. $t_c = \frac{l_c}{c}$, $t_c = \frac{5m}{3 \times 10^8\, m/c} = 1.67 \times 10^{-8}$ sec

$$l_c = \frac{\lambda^2}{\Delta\lambda}, \quad \Delta\lambda = \frac{\lambda^2}{l_c} = \frac{\left(1.5 \times 10^{-6}\, m\right)^2}{5m}$$

$\therefore \quad \Delta\lambda = 4.5 \times 10^{-13}$ m

Ans: **c**

9. $\vec{r} = |\vec{r}|\hat{\vec{r}} \Rightarrow \frac{d\vec{r}}{dt} = \frac{d|\vec{r}|}{dt}\hat{\vec{r}} + |\vec{r}|\frac{d\hat{\vec{r}}}{dt}$

If $|\vec{r}|$ is constant, then $\frac{d}{dt}|\vec{r}| = 0$ Then

$\frac{d\vec{r}}{dt} = |\vec{r}| e_r d\hat{\theta}$ means that the first time derivative is perpendicular to the vector.

As in circular motion with $|\vec{r}|$ constant the velocity is along tangent. However if the direction is also constant it will not be so. Further if direction is constant i,e fixed then $\frac{d\hat{\vec{r}}}{dt} = 0$ Then $\frac{d\vec{r}}{dt} = \frac{d|\vec{r}|}{dt}\hat{\vec{r}}$ means

$\frac{d\vec{r}}{dt} \,//\, \hat{\vec{r}}$ As motion in a straight line.

Ans: **a & c**

10. For an ideal gas under isothermal conditions PV = constant. Differentiating

$$PdV + VdP = 0 \Rightarrow -\frac{1}{V}\left(\frac{dV}{dP}\right)_T = \frac{1}{P}$$

In an adiabatic process $PV^{\gamma} = constant$ differentiating

$$P\gamma V^{\gamma-1}dV + V^{\gamma}dP = 0 \Rightarrow -\frac{1}{V}\left(\frac{dV}{dP}\right)_S = \frac{1}{\gamma P}$$

Ans: **b & d**

11. Miller Indices of triclinic crystal. The intercepts on three axes are $a, \frac{b}{2}, 3c$.

Dividing by primitive vectors we get $\frac{a}{a} = 1, \ \frac{b}{2}/b = \frac{1}{2} \ and \ \frac{3c}{c} = 3$ .Therefore the reciprocals being $\frac{1}{1} = 1, \ \frac{1}{1/2} = 2 \ and \ \frac{1}{3} = \frac{1}{3}$. Multiplying throughout by 3 we get $3, 6, 1$ hence the Miller Indices of the plane are (361)

Ans: **c**

12. The life time of the muon when moving with $v = .998c$ is $\tau = \frac{2.2 \times 10^{-6}}{\sqrt{1 - \frac{(0.998)^2 c^2}{c^2}}} = 34.80\ \mu s$

Thus in $34.80\ \mu s$ with speed 0.998c, the muon can travel a maximum distance

$$x = 0.998c \times 3 \times 10^8 \times 34.80 \times 10^{-6} = 10.42 Km$$

Muons travelling with this speed can reach earth surface. Of course they can reach earth if travel faster than this.

The apparent thickness of the atmosphere in the frame of muons is $\ell = \ell_\circ \sqrt{1 - \frac{v^2}{c^2}}$

$or \ \ell = 10.4\sqrt{1 - \frac{(0.998)^2 c^2}{c^2}} = 0.66\ km$ Thus

$\ell = 0.66 km \ and \ \ not \ 0.96 km$

Ans: **c, d**

13. From the theory of relativity, the energy of a particle is $E = \sqrt{p^2c^2 + m_0^2c^4}$ which turns to $E = pc$ if the rest mass $m_0 = 0$.

Only photon can run with speed of light and has zero rest mass

Ans: **b,c &d**

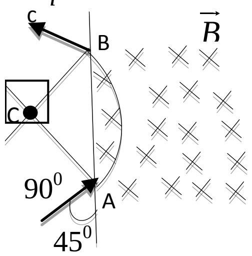
14. Diffraction takes place only when the size of obstacle is of the order of the wave length. In Fresnel diffraction experiments, distances need not be too large (infinitely), rather can be comparable to the wave - length of light. Also, there must be coherence in the source.

Ans :**a,c& d**

15. A charge particle entering a perpendicular magnetic field goes along a circular path, the necessary centripetal force being

$$\frac{mv^2}{R} = qvB \;\Rightarrow\; radius\;\; R = \frac{mv}{qB}$$

CA and CB are Perpendicular on Respective tangents.



Further both CA & CB are the two radii hence equal so angle ACB $= 90^0 = \frac{\pi}{2}$ this confirms that the path of charge particle from A to B is a quartier circle so time spent in the field region is

$$t = \frac{T}{4} = \frac{1}{4}\,2\pi\frac{m}{qB} = \frac{\pi m}{2qB}$$ The distance AB

is $= R\sin 45^0 + R\sin 45° = \frac{R}{\sqrt{2}} + \frac{R}{\sqrt{2}} = R\sqrt{2}$

If the direction of $\vec{B}$ is reversed the charge particle will traverse $\frac{3}{4}th$ of the circle and will come out so d is wrong

Ans : **a, b, & c**

16. The capacity of the capacitor is

$$C = \frac{\varepsilon_\circ A}{d} = \frac{\varepsilon_\circ \pi R^2}{d}$$ and the charge

$$q = CV = \frac{\varepsilon_\circ \pi R^2 V_\circ}{d}\sin\omega \mathrm{t}$$

The electric field between the plates is

$$E = \frac{V}{d} = \frac{V_\circ \sin\omega t}{d}$$

There by the displacement current density is $J_d = \varepsilon_0\frac{dE}{dt} = \frac{\omega\varepsilon_\circ V_\circ}{d}\cos\omega t$ further

$$\oint B.dl = \mu_\circ I_d = \frac{\mu_\circ\omega\varepsilon_\circ\pi R^2 V_\circ}{d}\cos\omega t$$ or

$$B = \frac{\mu_\circ\omega\varepsilon_\circ\pi R^2 V_\circ}{2\pi R d}\cos\omega t$$ which may not be the same everywhere between the plates.

Now the Pointing vector between the Plates $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_\circ}$ So the average value

$$< S > = \frac{E_0\sin\omega t \times B_0\cos\omega t}{\mu_\circ} = 0$$

Also curl B in question is calculated as

$$\vec{\nabla}\times\vec{B} = \mu_0 J_d = \frac{\mu_\circ\omega\varepsilon_\circ V_\circ}{d^2}\cos\omega t$$ Independent of r. The lines of B must be circular

Ans: **a, b& d**

17. The nuclear reaction $\pi^+ + n = \Delta^0 + K^+$ conserves Isospin, Strangeness and Baryon number and is carried through strong interaction.

Ans: **a, b, c & d**

18. The Compton shift

$$\Delta\lambda = \lambda^/ - \lambda = \frac{h}{m_0 c}(1-\cos\phi)$$ Also using

$c = \nu\lambda$ we get $\frac{c}{\nu^/} - \frac{c}{\nu} = \frac{h}{m_0 c}(1-\cos\phi)$

Given that $\nu^/ = \frac{\nu}{2}$, then we get

$$1-\cos\phi = \frac{m_0 c^2}{h\nu} \;\; or \;\; 1-\frac{m_0 c^2}{h\nu} = \cos\phi$$

Further the conservation of momentum perpendicular to direction of incidence gives $\frac{h\nu^/}{c}\sin\phi = mv\sin\theta \; or \; \frac{mv}{h\nu^/ / c} = \frac{\sin\phi}{\sin\theta}$

Ans: **a, c**

19. An emitter follower is a common collector amplifier with negative gain (less than 1) and with both the output and input in same phase

Ans: **b, c & d**

20. The dimensions are

$$\frac{L}{CR} = \frac{1}{CR}.\frac{L}{R}.R = \frac{1}{\sec ond}\times \sec ond \times\Omega \equiv \Omega$$

$\frac{E}{H} = characteristic\ impedance\ of\ free\ space \equiv \Omega$

$\frac{\mu}{\varepsilon} = \frac{Henery / meter}{second / \Omega \times meter} = \frac{Henery}{second} \times \Omega$

$= \frac{Henery}{\Omega} \times \frac{\Omega}{\sec} . \Omega = \Omega^2$

$\frac{B}{D} = \frac{weber / m^2}{coulomb / \mathrm{m}^2} = \frac{weber}{amp \times \sec} = \frac{\text{henery}}{\sec} = \Omega$

Ans: **a, b & d**

21. $E_2 - E_1 = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = hc\left(\frac{\lambda_2 - \lambda_1}{\lambda_1 \lambda_2}\right)$

$= hc \frac{300 \times 10^{-10}}{4950 \times 5250 \times 10^{-20}}$

$= \left(6.6 \times 10^{-34} \times 3 \times 10^8\right) \times 1.1544 \times 10^5$

$= 2.30 \times 10^{-20} J$ so option b is correct.

Ans: **b**

22. Energy released in one reaction

$= 3 \times 2.014 - \left(4.001 + 1.077 + 1.008\right) = 0.026\ u$

So energy released by 3 deuterons $= 0.026\ u \times 931.5 = 24.219 MeV$.

So the energy released by 10^{40} deuterons shall be $= 8.073 \times 10^{40} MeV$

$= 8.073 \times 10^{40} \times 10^6 \times 1.6 \times 10^{-19}\ joule$

$= 1.29 \times 10^{28} J$ This energy can last for a

Time $t = \frac{1.29 \times 10^{28}}{10^{16}} = 1.29 \times 10^{12}$ second

Also $t = 4.1 \times 10^4$ year

Ans: **b, c**

23. In a p-n junction the current mechanism is diffusion in forward bias and drift in reveres bias

Ans: **b**

24. According to uncertainty principle

$\Delta x \times \Delta p = \frac{\hbar}{2} = \frac{h}{4\pi}$ given that $\Delta x = 0.5 A^\circ$

if there by the uncertainty in momentum is $\Delta p = \frac{h}{4\pi \times 0.5 \times 10^{-10}} = \frac{h}{2\pi} \times 10^{10}$ ans d

Also according to Bohr atom model

$mvr = n\frac{h}{2\pi}$ therefore the radius of second orbit is

$r = 2 \times \frac{h}{2\pi \times momentum}$

$= 2 \times \frac{h}{2\pi \times \frac{h}{2\pi} \times 10^{10}} = 2 A^\circ$

Ans: **b**

25. If D denotes the doping concentration in a semiconductor, then it is known that the depletion width $x \propto \frac{1}{\sqrt{D}}$ and the diffusion length $L \propto \sqrt{D}$

Ans: **b**

PART B1

B1. Refuted since the wavelength of micro waves (frequency $\approx 10^9 hz\ to\ 10^{12} hz$) is often of the order of millimeter or centimetre and not micrometre.

B2. Refuted. There will be a phase change of π at the two surfaces hence constructive interference in reflected light.

B3. Defended. Without rotating the apparatus, one would not have reached the final results of the experiment which ultimately led to the conclusion that the speed of light in vacuum is a universal constant. A postulate of the special theory of relativity would not have come.

B4. Defended since the condition of diffraction is $\left(e + d\right)\sin\theta = n\lambda$

$\Rightarrow \frac{2.54}{8000}\sin 90 = n \times 625 \times 10^{-7} \Rightarrow n = 5$

B5. Defended. In a uniaxial crystal the O-ray and E-ray travel with different velocities in different directions. The difference in the two velocities being zero along the optic axis while a maximum in a direction perpendicular to the optic axis.

B6. The precessional frequency also known as Larmor frequency refers to the rate of precession of a magnetic dipole around the direction of an external magnetic field and isexpressed as $\omega_p = \gamma B = \frac{qB}{2m}$ (often discussed in vector atom model) here γ is the gyromagnetic ratio. The cyclotron frequency $f_c = \frac{qB}{2\pi m}$ denotes the number of circulations of a proton in a perpendicular magnetic field B.

B7. Defended. For pure rolling of a solid spherical ball acceleration $\vec{a} = \vec{\alpha} \times \vec{R}$ (must be). If the force imparted by the que to the billiard ball is F and the ball is hit a distance h above the central line then

$$h \times F = I\alpha \;\; or \;\; hF = \frac{2}{5} mR^2 \alpha$$

Now using $F = ma \, and \, a = \alpha R$

we get $hma = \frac{2}{5} mR^2 \frac{a}{R} \Rightarrow h = \frac{2}{5} R$ Hence the result

B8. Since the luminosity of star is 17000 time that of the Sun therefore

$$\sigma T^4 = 17000\sigma (5800)^4 \Rightarrow T = 66228k \cong 66000K$$

B9. For an electron gas in a metal, the number of free electrons is

$$n(E) = \int_0^{E_f} g(E)dE = \frac{8\sqrt{2m}\pi Vm}{h^3} \int_0^{E_f} E^{\frac{1}{2}} dE \text{ or}$$

$$n = \frac{16\sqrt{2}\pi V}{3} \left(\frac{m}{h^2}\right)^{\frac{3}{2}} E_f^{\frac{3}{2}}$$

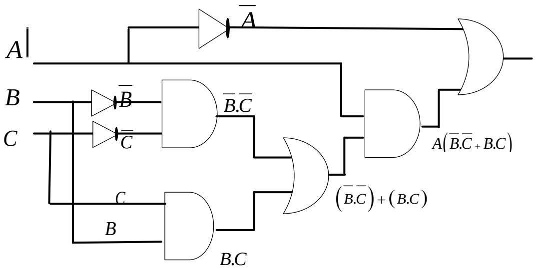
Or $E_f = \frac{h^2}{2m} \left(\frac{3n}{8\pi V}\right)^{\frac{2}{3}}$ or E_f depends on the electron density $\left(\frac{n}{V}\right)$. It may have same value even when n and V changes but $\left(\frac{n}{V}\right)$ remains same hence justified

B10. Truth table:

Boolean Expression: $Y = \bar{A} + A\left(BC + \bar{B}\bar{C}\right)$

A	B	C	D
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

The Boolean diagram is



Solutions: Part B – 2

P1. (a) The diagonals of a cube form an isosceles triangle with base angle $\beta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$. Obviously, the acute angle between the diagonals will be $\theta = (180 - 2\beta)$ So

$$\sin\theta = \sin(180 - 2\beta) = \sin 2\beta = 2\sin\beta\cos\beta$$

$$\sin\theta = 2\frac{1}{\sqrt{3}} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{2\sqrt{2}}{3} \Rightarrow \cos\theta = \frac{1}{3}$$

(b)Knowing that $\vec{v} = \vec{\omega} \times \vec{r}$ or

$$\vec{r} \times \vec{v} = \vec{r} \times \left(\vec{\omega} \times \vec{r}\right) = r^2 \vec{\omega} \;\; \Rightarrow$$

$$\vec{\omega}=\frac{\vec{r}\times\vec{v}}{|\vec{r}|^2}=\frac{1}{\left(\sqrt{1^2+9^2+8^2}\right)^2}\begin{pmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 9 & -8\\ 3 & -4 & 5\end{pmatrix}$$

Or $\vec{\omega}=\frac{1}{146}\left\{13\hat{i}-29\hat{j}-31\hat{k}\right\}$

The angular momentum of the particle may be

$$L=r\times p=\left(\hat{i}+9\hat{j}-8\hat{k}\right)\times 2\left(3\hat{i}-4\hat{j}+5\hat{k}\right)$$

or $L=2\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 9 & -8\\ 3 & -4 & 5\end{vmatrix}$ or

$$L=(45-32)\hat{i}+(-24-5)\hat{j}+(-4-27)\hat{k}$$

Or $L=13\hat{i}-29\hat{j}-31\hat{k}$ kg m^2s^{-1}

P2. When a mechanical system, capable of oscillation, is subjected to a periodic force say $F=F_0 e^{\pm ipt}$ whose frequency is $\frac{p}{2\pi}$, the system starts oscillation. The equation of motion may be written

as $m\frac{d^2x}{dt^2}=-kx-r\frac{dx}{dt}+F_0e^{\pm ipt}$ or

$m\frac{d^2x}{dt^2}+r\frac{dx}{dt}+kx=F_0 e^{\pm ipt}$ this is the differential equation offorced oscillations of the system. Here r is the mechanical resistance and K is the force constant.

The equation may be rewritten as

$$\frac{d^2x}{dt^2}+\frac{r}{m}\frac{dx}{dt}+\frac{k}{m}x=\frac{F_0}{m}e^{\pm ipt} \text{ or}$$

$$\frac{d^2x}{dt^2}+2b\frac{dx}{dt}+\omega^2x=f_0 e^{\pm ipt}$$

The solution of this differential equation consists of two parts namely

(i) The complimentary function as

$x=Ce^{-bt}\sin(\beta t+\phi)$ with $\beta=\sqrt{\omega^2-b^2}$ and

(ii) The particular integral as

$x=\frac{f_0}{\sqrt{\left(p^2-\omega^2\right)^2+2b^2p^2}}e^{\pm i(pt+\theta)}$ The complete solution forforced oscillations may thus be

$$x=\frac{f_0}{\sqrt{\left(p^2-\omega^2\right)^2+2b^2p^2}}e^{\pm i(pt+\theta)}+Ce^{-bt}\sin(\beta t+\phi)$$

Where $\frac{\beta}{2\pi}+\frac{1}{2\pi}\sqrt{\omega^2-b^2}$ is the frequency of damped oscillations which die quite soon and final solution remains as

$x=\frac{f_0}{\sqrt{\left(p^2-\omega^2\right)^2+2b^2p^2}}e^{\pm i(pt+\theta)}$ thereby

the velocity at any time t is given by

$$v=\frac{dx}{dt}=\pm ip\frac{f_0}{\sqrt{\left(p^2-\omega^2\right)^2+2b^2p^2}}e^{\pm i(pt+\theta)}$$

substituting now the values, we get

$$\frac{dx}{dt}=\pm ipx=\frac{\pm ipF_0/m}{\sqrt{\left(p^2-\frac{k}{m}\right)^2+2\frac{r^2}{4m^2}p^2}}e^{\pm i(pt+\theta)}$$

$$=\pm\frac{F_0}{\frac{m}{p}\sqrt{\left(p^2-\frac{k}{m}\right)^2+2\frac{r^2}{4m^2}p^2}}e^{\pm i(pt+\theta)}$$

$$=\pm\frac{F_0}{\sqrt{\left(mp-\frac{k}{p}\right)^2+r^2}}e^{\pm i(pt+\theta)}=\frac{F_0}{Z_m}e^{\pm i(pt+\theta)}$$

Where $Z_m=\sqrt{r^2+\left(mp-\frac{k}{p}\right)^2}$ is the mechanical impedance.The mechanical impedance is the effective hurdle to the vibratory motion and seems to be quite similar to the electrical impedance with its resistive and reactive components.

P3. The given process is $P(V-nb)=nRT$ for n mole of the gas. Forone mole at temperature T_1 we can write

$$P_1(V_1-b)=RT_1$$

$$\Rightarrow R=\frac{P_1(V_1-b)}{T_1}$$

Also when one mole of the gas is heated from T_1 *to* T_2 at constant pressure

$$\frac{P_1(V_1-b)}{T_1}=\frac{P_1(V_2-b)}{T_2}=R$$

$\Rightarrow V_2=\frac{T_2}{T_1}(V_1-b)+b$ Further since the pressure is constant during heating,

the work done $W=\int_{V_1}^{V_2}PdV=P_1(V_2-V_1)$

$$=P_1\left[\frac{T_2}{T_1}(V_1-b)+b-V_1\right]$$

$$=P_1\left[\frac{T_2}{T_1}(V_1-b)-(V_1-b)\right]$$

$$=P_1(V_1-b)\left(\frac{T_2}{T_1}-1\right)=\frac{P_1}{T_1}(V_1-b)(T_2-T_1)$$

$$W=R(T_2-T_1)$$

Further the change in internal energy

$$dU=\int_{T_1}^{T_2}C_v dT=n\int_{T_1}^{T_2}(C_0-C_1T)dT$$

$$dU=nC_0(T_2-T_1)-\frac{1}{2}nC_1\left(T_2^2-T_1^2\right)$$

$dU=n(T_2-T_1)\left\{C_0-\frac{1}{2}C_1(T_2+T_1)\right\}$ For one mole gas

$$dU=(T_2-T_1)\left\{C_0-\frac{1}{2}C_1(T_2-T_1)\right\}$$

Now $dQ=dU+dW$

$$=(T_2-T_1)\left\{C_0-\frac{1}{2}C_1(T_2-T_1)\right\}+R(T_2-T_1)$$

$$dQ=(T_2-T_1)\left\{C_0+R-\frac{1}{2}C_1(T_2-T_1)\right\}$$

P4. Let E_y & B_z represent the electric and magnetic vectors of the plane polarised electromagnetic wave. Then the energy flux is given by the Poynting vector $\vec{P}=\frac{\vec{E}\times\vec{B}}{\mu_\circ}$ =

$$\frac{E_y\times B_Z}{\mu_\circ}\hat{i}=\frac{E_y B_Z}{\mu_\circ}\hat{i}$$

The energy density in electromagnetic field is

$$U=\frac{1}{2}\varepsilon_\circ E^2+\frac{1}{2\mu_\circ}B^2=\varepsilon_\circ E^2 \quad \because \frac{E}{c}=B$$

Further since the averageenergy in electric field = average energy in magnetic fieldi,e

$$\frac{1}{4}\varepsilon_\circ E_y^2=\frac{1}{4\mu_\circ}B_z^2 \Rightarrow B_z=\sqrt{\mu_\circ\varepsilon_\circ}\ E_y$$

Thus the energy flux

$$\frac{E_yB_z}{\mu_\circ}=\frac{E_y}{\mu_\circ}\sqrt{\mu_\circ\varepsilon_\circ}\,E_y \quad E_y=\frac{\varepsilon_\circ E_y^2}{\sqrt{\mu_\circ\varepsilon_\circ}}=cU$$

P5. The resultant intensity as a result ofdiffraction by N parallel slits i,e diffraction grating is $I=\left(\frac{A\sin\alpha}{\alpha}\right)^2\left[\frac{\sin N\beta}{\sin\beta}\right]^2$

Since $_{Lt\beta\to\pm n\pi}\left[\frac{\sin N\beta}{\sin\beta}\right]^2=N^2$ The

Maximum Intensity is $I=\left(\frac{A\sin\alpha}{\alpha}\right)^2N^2$

and the condition of Maximum is $\beta=n\pi$ or $(e+d)\sin\theta=n\lambda$ where

$n=0,\pm1,\pm2,\pm3\, and\ so\ on$ This is the condition of Principal maxima

The condition of minima is

$N(e+d)\sin\theta=m\lambda$ where

$m=1,2,3.............(N-1),(N+1),(N+2).....$

but $m\neq 0,N,2N,3N......$

See standard text: Principles of Optics by B K Mathur

P6. The charge moving in perpendicular magnetic field will go along a circular path of radius $R=\frac{mv_\circ}{qB_\circ}$ and will traverse a circle within the field region.Flux enclosed will be

$$\Phi=\pi R^2B_\circ=\pi\left(\frac{mv_\circ}{qB_\circ}\right)^2B_\circ=\pi\left(\frac{mv_\circ}{q}\right)^2\frac{1}{B_\circ}$$

The angular momentum $=mvR$ is obtained as $\Rightarrow L=qB_\circ R^2$

The magnetic dipole moment

$$\mu=iA=nq\pi R^2=\frac{\omega}{2\pi}q\pi R^2=\frac{qv_0R}{2}$$

Thus $\frac{\mu}{L}=\frac{qv_\circ R}{2}/\mathrm{m}v_0R=\frac{q}{2m}$

P7. The particle wave function is given as $\psi(x,0)=\frac{1}{\sqrt{2}}\{\phi_\circ(x)+\phi_1(x)\}$ The energy Eigen valuesmay be calculated as

$$<H(x,0)>=\int_0^L \psi(x,0)\left(-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right)\psi(x,0)dx$$

$$\frac{1}{\sqrt{2}}\int\{\phi_\circ^*(x)+\phi_1^*(x)\}\{E_\circ\phi_\circ(x)+E_1\phi_1(x)\}dx$$

$$=\frac{1}{2}(E_\circ+E_1)=\frac{1}{2}\left(\frac{\pi^2\hbar^2}{2mL^2}+\frac{2^2\pi^2\hbar^2}{2mL^2}\right)$$

$$=\frac{5}{2}\left(\frac{\pi^2\hbar^2}{2mL^2}\right)=\frac{5}{4}\frac{\pi^2\hbar^2}{mL^2}$$

P8. (a) The value of

$$\frac{e^2}{4\pi\varepsilon_\circ}=\left(1.602\times10^{-19}\right)^2\times8.99\times10^9 \text{ N } m^2$$

$$=\left(1.602\times10^{-19}\right)^2\times8.99\times10^9 \text{ Joule x meter}$$

$$=\frac{\left(1.602\times10^{-19}\right)^2\times8.99\times10^9}{1.602\times10^{-19}}eV\times10^{15}\,fm$$

$$=\frac{1.602\times10^{-19}\times8.99\times10^9}{10^6}\times10^{15}\,MeVfm$$

$$=1.602\times8.99\times10^{-19+9+15-6}\,MeV\ fm$$

$$\frac{e^2}{4\pi\varepsilon_\circ}=1.602\times0.899MeVfm=1.44MeVfm$$

Now the fine structure constant $\alpha=\frac{1}{4\pi\varepsilon_\circ}\frac{e^2}{\hbar c}$

$$\alpha=\frac{8.99\times10^9\times\left(1.602\times10^{-19}\right)^2}{197.3386}\frac{Newton\times m^2}{MeV\times fm}$$

$$=\frac{1.44}{197.3386}\frac{MeVfm}{MeVfm}$$

$$\alpha=7.2971\times10^{-3}=\frac{1}{137.0407}\approx\frac{1}{137}$$

(b) (i) The most probable state corresponds to the most probabledistribution of the particles in various cells. It can be proved that the most probable distribution is obtained when the number of particles is equally distributed in all the cells.

$\therefore$ For most probable distribution, the number of particles in each cell $=\frac{n}{G}$

Where $n=8$ is the total number of particles and $G=g_1+g_2$ the total number of cells in allthe k compartments.

Number of particles in the ith compartment $=\frac{n}{G}g_i$ where g_i is the number of cells in the ithcompartment.

Number of particles, $n=8$

Number of cells in compartment 1, $g_1=4$

Number of cells in compartment 2, $g_2=2$

Total number of cells in all the (two) compartment $G=g_1+g_2=6$

$\therefore$ For most probable distribution, the number of particles in compartment one $=\frac{8}{6}\times4=\frac{16}{3}=5\frac{1}{3}$ and in the compartment two $=\frac{8}{6}\times2=\frac{8}{3}=2\frac{2}{3}$

As fractions of particles are not possible, 2 distributions are equally most probable i.e. $(6,2)$ and $(5,3)$

$$\therefore W(6,2)=\frac{8!}{6!2!}(4)^6(2)^2=28\times2^{14}=7\times2^{16}$$

$$\therefore W(5,3)=\frac{8!}{5!3!}(4)^5(2)^3=56\times2^{13}=7\times2^{16}$$

(ii) The probability of macrostate $(8,0)$ is

$$W(8,0)=\frac{8!}{8!\times(8-8)!}(4)^8(2)^0=4^8=2^{16}$$

P9. The loop of edge length a, is moving in the magnetic field $\therefore$ inducedemf = $l\,B\,v=a\,B\,\omega\,r$

And the induced current is = $\frac{\text{Induced emf}}{R}$

$i=\frac{a\,B\,\omega\,r}{R}$ where R is the resistance

On length a, the force is $=ilB=iaB$

$$F=\frac{aB\omega r}{R}aB=\frac{a^2B^2\omega r}{R}$$

The torque of this force F is $r\times F$

$$\tau=\frac{a^2B^2\omega r}{R}r=\frac{a^2B^2\omega r^2}{R}$$

Now $R = \frac{\rho l}{A} = \frac{1}{\sigma}\frac{a}{at} = \frac{1}{\sigma t}$ Substituting

$$\tau = \frac{a^2B^2\omega r}{\frac{1}{\sigma t}} = a^2B^2\omega r^2\sigma t$$

P10. (a) Knowing that if a number is expressed in base r or redix r, then

$$a_n r^n + a_{n-1}r^{n-1} + a_{n-2}r^{n-2} ..... + a_2r^2 + a_1 r + a_0 = 0$$

in the present case the equation is

$x^2 - 10x - 31 = 0$ *so* $a_2 = 1, a_1 = -10$ *and* $a_0 = 31$

now $x = 5 \Rightarrow 5^2 - 10_r \times (5) + 31_r$ where

$10r = r$ *and* $31r = 3 \times r + 1$

So $25 - 5r + 3r + 1 = 0 \Rightarrow r = 13$

And $x = 8 \Rightarrow 8^2 - 10_r \times (8) + 31_r = 0$ where

$10r = r$ *and* $31r = 3r + 1$ so

$64 - 8r + 3r + 1 = 0 \Rightarrow 5r = 65$ *or* $r = 13$

Thus the basis of numbers is 13

(b) For a cubic crystal structure with lattice parameter a, the spacing d between planes is given by $d = \frac{a}{\sqrt{h^2+k^2+l^2}}$

Now knowing that $2d\sin\theta = n\lambda$ we get

$\sin\theta = \frac{n\lambda}{2d} = \frac{n\lambda}{2a}\sqrt{h^2+k^2+l^2}$ or

$\sin^2\theta = \frac{n^2\lambda^2}{4a^2}(h^2+k^2+l^2)$ as h, k, l are all positive integers, for any pair of diffraction lines one can write $\frac{\sin^2\theta_1}{\sin^2\theta_2} = \frac{h_1^2+k_1^2+l_1^2}{h_2^2+k_2^2+l_2^2}$

Conditions for FCC: All (h, k, l) odd or all even

BCC: Sum of h, k, l is even

Primitive cubic: none of above

See table below: The lattice parameter can be calculated using any set of ($h\,k\,l$)

$$a = \left(\frac{\lambda}{2\sin\theta}\right) \times \sqrt{h^2+k^2+l^2}$$

$$a = \left(\frac{0.154\,nm}{2 \times 0.37298}\right) \times \sqrt{3} = 0.2066 \times 1.73 = 0.3575\,nm$$

Thereby the radius of copper atom is

$$R = \frac{a\sqrt{2}}{4} = \frac{a}{2\sqrt{2}} = \frac{0.3575}{2\sqrt{2}} = 0.1265\,nm$$

Peak	2θ	$\sin^2\theta$	$\sin^2\theta_1/\sin^2\theta_2$	K factor	$h^2+k^2+l^2$	$h\,k\,l$	$\sqrt{h^2+k^2+l^2}$	d(nm)	R(nm)
1	43.8	0.139119	1	3	3	111	1.7320	0.2066	0.1265
2	50.8	0.183985	1.3225	3.9675	4	200	2.0000	0.1790	0.1271
3	74.4	0.365540	2.6275	7.8825	8	220	2.8284	0.1265	0.1275
4	90.4	0.503490	3.61913	10.8573	11	311	3.3166	0.1078	0.1274

$$S = \frac{1}{\mu_\circ}\frac{\mu_\circ \omega \varepsilon_\circ \pi R^2 V_\circ}{2\pi R d}\cos\omega t \times \frac{V_\circ \sin\omega t}{d} = \frac{\omega\varepsilon_\circ R}{4d^2}V_\circ^2 \sin 2\omega t$$

The maximum value of Pointing vector will therefore be $S_{\max} = \frac{\omega\varepsilon_\circ R}{4d^2}V_\circ^2$