# INDIAN ASSOCIATION OF PHYSICS TEACHERS

## National Graduate Physics Examination 2020

**Day and Date of Examination : Sunday, January 19, 2020**

**Time : 10 AM to 1 PM**

***Instructions to Candidates***

1. In addition to this question paper, you are given **answer sheet (OMR Sheet) for part A** and **answer paper for part B**.
2. On the answer sheet (OMR Sheet) for part A, fill up all the entries carefully in the space provided, **Only in block capital. Do write the name and PIN of your city.**

   **Incomplete / incorrect / carelessly filled information may disqualify your candidature**
3. On part A answer sheet, use only BLUE or BLACK BALL PEN for making entries and marking answers.
4. In Part A each question has **FOUR** alternatives. Any number of these (4, 3, 2 or 1) may be correct. You have to mark **ALL** correct alternatives and fill a bubble (●) for each, like

| Q.No. | a | b | c | d |
|---|---|---|---|---|
| 24 | ○ | ● | ○ | ● |

   Full marks are 6 for each question, you get them only when ALL correct answers are marked. The answers of part A shall be available on **www.indapt.org.in** on 1.2.2020.
5. Part A answer sheet will be collected at the end of one hour.
6. Any rough work should be done only on the sheets provided with part B answer paper.
7. Use of non-programmable calculator is allowed.
8. No candidate should leave the examination hall before the completion of the examination. You will take away the question paper with you.
9. Symbols used in the paper have their usual meaning unless specified otherwise.

***PLEASE DO NOT MAKE ANY MARK OTHER THAN ● IN THE SPACE PROVIDED ON THE ANSWER SHEET OF PART A***

Answer sheets for part A are to be evaluated with the help of a machine. Due to this, **CHANGE OF ENTRY IS NOT ALLOWED**

**Scratching or overwriting may result in wrong score**

**DO NOT WRITE ANYTHING ON BACK SIDE OF ANSWER SHEET FOR PART A**

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## National Graduate Physics Examination 2020

**Day and Date of Examination : Sunday, January 19, 2020**

**Time : 10 AM to 1 PM**

Part A- Maximum Marks: 150 — Time for Part A : 60 minutes

Part B- Maximum Marks: 150 — Time for Part B : 120 minutes

## Part A

25 x 6 = 150

**Mark the correct option/options (Any number of options may be correct).**
**Marks will be awarded only if all the correct options are marked. No negative marking.**

1. Unit Vector $\hat{r} = \frac{1}{|\vec{r}|}\left(\hat{i}x + \hat{j}y + \hat{k}z\right)$

$$\vec{\nabla}\cdot\hat{r} = \vec{\nabla}\cdot\frac{1}{|\vec{r}|}\left(\hat{i}x + \hat{j}y + \hat{k}z\right)$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\frac{1}{\sqrt{x^2+y^2+z^2}}\left(\hat{i}x + \hat{j}y + \hat{k}z\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right) + \frac{\partial}{\partial z}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$$= \frac{3}{\left(\sqrt{x^2+y^2+z^2}\right)} - \frac{x^2+y^2+z^2}{\left(x^2+y^2+z^2\right)^{3/2}} = \frac{3}{r} - \frac{1}{r} = \frac{2}{r}$$

Ans: c

2. In the system of cylindrical coordinates the volume element is $d\tau = s\,ds\,d\phi\,dz$

Ans: d

3. The electric field is $E = ax\hat{i} + cz\hat{j} + 6by\hat{k}$ the electric field must be conservative hence $\vec{\nabla} \times \vec{E}$ must vanish so

$$\vec{\nabla}\times E = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ax & cz & 6by \end{vmatrix} = 0$$

$$\hat{i}\left\{\frac{\partial}{\partial y}6by - \frac{\partial}{\partial z}cz\right\} + \hat{j}\left\{\frac{\partial}{\partial z}ax - \frac{\partial}{\partial x}6by\right\} + \hat{z}\left\{\frac{\partial}{\partial x}cz - \frac{\partial}{\partial y}ax\right\} = 0 \text{ Or}$$

$i\,(6b-c) + 0 + 0 = 0$ (must be) so the answer is c.

Ans: c

4. In Newton's ring experiment with lens of large radius of curvature R, one can write $2\mu t = n\lambda$ when lens is raised by h $2\mu(t+h) = (n+1000)\lambda$ Using $\mu = 1$ for air, one obtains $\lambda = \frac{h}{500}$

Ans. c

5. The wave function f (x) must be
   a) Continuous
   b) Single valued and
   c) Differentiable

All these properties are exhibited by the function in figure (a)

Ans: a

6. Knowing that magnetic induction $\vec{B} = \vec{\nabla} \times \vec{A}$ Using stokes' theorem

$$\oint \vec{A}\cdot d\vec{l} = \oiint\left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{S} = \oiint \vec{B}\cdot d\vec{S} = \phi_B$$

Ans. c

7. According to uncertainty principle $\Delta p_y\,\Delta y = \hbar$

$$\therefore \Delta p_y = \frac{\hbar}{\Delta y} = \frac{6.63\times10^{-34}}{2\pi\times1\times10^{-9}}$$

$$\therefore \Delta p_y = 1.055\times10^{-25} = 1.06\times10^{-25}\ \frac{\text{kgm}}{\text{sec}}$$

Also $\Delta v_y\,\Delta y = \frac{\hbar}{m}$ or

$$\Delta v_y = \frac{h}{2\pi m\,\Delta y} = \frac{6.63\times10^{34}}{2\pi\times9.1\times10^{-31}\times1\times10^{-9}}$$

$$\therefore \Delta V_y = 1.16\times10^{5}\ \frac{m}{\text{sec}}$$

Ans: a, c

8. Total charge in the thick spherical shell between is r = a and r = b is

$$q=\int_a^b \frac{k}{r^2}4\pi r^2\,dr = 4\pi k\int_a^b dr$$

$\therefore$ q = 4 $\pi$ k (b - a) Then

$$\therefore \vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{4\pi k\,(b\text{-}a)}{r^2}\hat{r} \Rightarrow \vec{E}=\frac{k\,(b\text{-}a)}{\varepsilon_0 r^2}\hat{r}$$

Ans: b

9. If the fourth quadrant in added, the centre of mass should go back to O. So if OP = z then

$$\frac{\frac{3M}{4}z-\frac{M}{4}\times\frac{a}{2\sqrt{2}}}{M}=0 \Rightarrow z=\frac{a}{6\sqrt{2}}$$

Ans: a

10. The intensity at a principal maximum is

$$I=\left(\frac{A\sin\alpha}{\alpha}\right)^2(N\tan N\beta)^2$$

with all the standard symbols. Further

Lt $\beta\to\pm n\pi$, $(N\tan N\beta)^2 = N^2$ Therefore

$$I=\left(\frac{A\sin\alpha}{\alpha}\right)^2(N)^2$$ the first factor decreases

in higher orders of diffraction

Ans: b, d

11. The force F is conservative if $\vec{\nabla}\times\vec{F}=0$ or

$\vec{F}(x)=\left(\frac{1}{x^2}-\frac{x^2}{2}\right)i$; $\vec{\nabla}\times\vec{F}=0$ Hence $\vec{F}(x)$

is conservative. The particle is in equilibrium when $F(x)=0\Rightarrow x^4=2\Rightarrow x=2^{\frac{1}{4}}\approx 1.18$

At x = 1, F (x) is positive, so particle moves towards $x=\sqrt{2}$

Ans: a, c

12. When a beam of plane polarised light passes through quarter $\left(\frac{\lambda}{4}\right)$ wave plate. The emergent light may be plane polarised, circularly or even elliptically polarised depending upon how the optic axis is inclined with respect to the direction of $\vec{E}$ vector of the incident light.

Ans: b, c, d

13. Sudden withdrawal of magnetic material from magnetic field under adiabatic conditions causes a change of temperature due to increase in magnetic component of entropy.

Ans. a

14. Miessner effect can be used to distinguish between supper conductor and perfect metal

Ans: c

15. Nano rods are one-dimensional objects hence considering one dimensional motion of the conduction electrons in these rods; we can apply the same concepts to calculate the density of states as for a 1 - D free electron gas:

Using periodic boundary conditions, one may have $\psi(x+L)=\psi(x)$ ................ 1

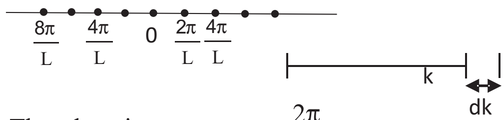
Where L= length of the Nano rod. Now using

$\psi(x)=Ae^{ikx}$ .............. 1 (for a free electron)

One gets using (1) & (2)

The allowed values of k are

$$k = 0, \pm\frac{2\pi}{L}, \pm\frac{4\pi}{L}, .......$$ And so on




Thus there is one state per $\frac{2\pi}{L}$

Therefore the number of states between k and k + dk is = $\frac{2dk}{\frac{2\pi}{L}}=\frac{L}{\pi}dk$

Taking D (k) as the density of states (DOS) by definition one can write $D(k)\,dk=\frac{L}{\pi}dk$

Now using $E=\frac{\hbar^2k^2}{2m}$ & $dE=\frac{\hbar^2}{m}k\,dk$

or $dk=\frac{m}{\hbar^2}\frac{1}{k}dE$ and further writing

D (k) dk - D (E) dE

= number of states between k and (k + dk)

= number of states between E and (E + dE)

We get;

$$D(E)\,dE=\frac{L}{\pi}dk=\frac{L}{\pi}\frac{m}{\hbar^2}\frac{1}{\sqrt{\frac{2mE}{\hbar^2}}}dE$$ or

$$D(E)\,dE=\frac{L}{\pi}\frac{\sqrt{m}}{\hbar\sqrt{2}}\cdot\frac{1}{\sqrt{E}}dE\Rightarrow D\ (E)\propto\frac{1}{\sqrt{E}}$$ or

$$D(E)\propto E^{-\frac{1}{2}}$$

Ans: c

16. Heat given = Heat taken so

$$c_1(T_f - T_1) = c_2(T_2 - T_f)$$

Or $T_2 = \frac{c_1}{c_2}(T_f - T_1) + T_f$

And the net change in entropy is

$$s = c_1\int_{T_1}^{T_f}\frac{dT}{T} + c_2\int_{T_f}^{T_2}\frac{dT}{T} = c_1 \ln\frac{T_f}{T_1} + c_2 \ln\frac{T_2}{T_f}$$

$$s = \ln\left(\frac{T_f}{T_1}\right)^{c_1} + \ln\left(\frac{T_2}{T_f}\right)^{c_2} = \ln\left(\frac{T_f^{c_1}}{T_1^{c_1}} \times \frac{T_2^{c_2}}{T_f^{c_2}}\right)$$

$$s = \ln\left(\frac{T_2^{c_2}}{T_1^{c_1}} \cdot T_f^{(c_1 - c_2)}\right)$$

Ans: a, c

17. Two SHM are x = a and sin ωt and

$y = a\sin\left(2\omega t + \frac{\pi}{2}\right)$ Thereby

$y = a\cos 2\omega t$

$= a\left[1 - 2\sin^2\omega t\right] = a\left[1 - 2\frac{x^2}{a^2}\right]$ Or

$y = a - \frac{2x^2}{a}$ Or $x^2 = -\frac{a}{2}(y - a)$

This is the equation of parabola.

Ans: c

18. (Magnetic force = Electric force)

$F_{mag} = F_{el}$

$\frac{F_{mag}}{l} = \frac{\mu_0}{4\pi}\frac{2i_1 i_2}{d}$ Where $i_1 = \lambda v$ and $i_2 = \lambda v$

$$\frac{F_{mag}}{l} = \frac{\mu_0}{4\pi}\frac{i_1 i_2}{d} = \frac{\mu_0}{2\pi}\frac{\lambda^2 v^2}{d} \quad \text{...... (1)}$$

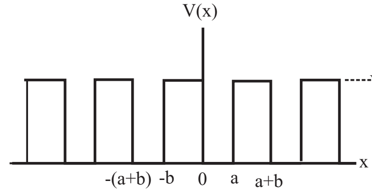
$$\frac{F_e}{l} = \frac{\lambda}{2\pi\varepsilon_0 d} \times \lambda = \frac{\lambda^2}{2\pi\varepsilon_0 d} \quad \text{...... (2)}$$

$$\therefore \frac{\mu_0}{4\pi}\frac{2 \times \lambda^2 \times v^2}{d} = \frac{\lambda^2}{2\pi\varepsilon_0 d}$$

$$v^2 = \frac{1}{\mu_0\varepsilon_0} \Rightarrow v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c \therefore v = c$$

Ans: d

19. Kronig and Penney model is a simplified model for an electron being in a one dimensional periodic potential. In this model, the potential V(x) is a periodic square wave where the electron experiences an infinite one dimensional array of finite potential wells as shown in figure. Each potential well models attraction to an atom in the lattice. The width of the wells must correspond roughly to the lattice spacing.



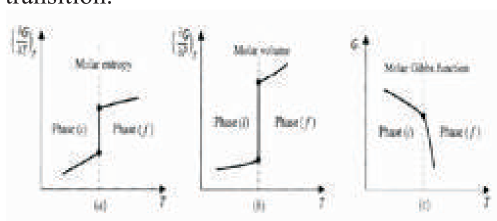

Ans : b

20. The energy states of a particle in a one-dimensional quantum well are

$E_n = \frac{n^2\hbar^2}{2ma^2} = 4.4\ eV$ when the width of well is doubled a is replaced by 2a, the energy is decreased four times and becomes equal to 1.1 eV.

Ans: a

21. Characteristics of a first-order phase transition are the discontinuous changes in (a) molar entropy and (b) molar volume where as the (c) Gibbs function is single valued with a discontinuous slope as shown below. Latent Heat is essentially involved in the first-order transition.

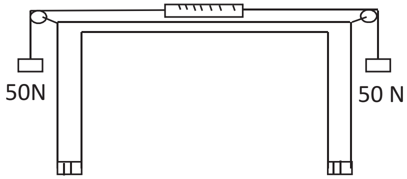


Ans: a, b, c & d

22. Fermions are indistinguishable, have half odd integral spin $\approx (2n+1)\frac{\hbar}{2}$ and obey Pauli Exclusion Principle. The wave function is anti-symmetrical. Boson, are indistinguishable, have integral spin ≈ $n\hbar$ and do not obey Pauli Exclusion Principle. The boson wave function is always symmetrical.

Ans: a, b, d

23. The tension in the spring shall be 50 N




Ans: b

24. $b_n = \frac{2}{L}\int_O^L f(x)\sin\frac{n\pi x}{L}dx = \frac{2}{L}\int_O^L x\ \sin\frac{n\pi x}{L}dx$

$$b_n = \frac{2}{\not{L}}\frac{\not{L}}{n\pi}\left\{-x\cos\frac{n\pi x}{L}\Big|_0^L - \int_0^L \cos\frac{n\pi x}{L}dx\right\}$$

$$b_n = \frac{2}{n\pi}\left[L\cos n\pi + 0\right] - \frac{2}{n\pi}\frac{L}{n\pi}\left[\sin\frac{n\pi x}{L}\Big|_0^L\right]$$

$$\therefore\ b_2 = \frac{\not{2}L}{\not{2}\pi}\cos 2\pi = \frac{L}{\pi}\quad for\ n = 2$$

The value of the coefficient $b_2 = \frac{L}{\pi}$

Ans: b

25. The Boolean expression for the output of the circuit is $Q = \overline{A\cdot B + C\cdot D}$

Ans: a

## Part B1 (Short Answer Question)

B1. If at all, a uniform magnetic field is produced, in a cubical region, along Z-axis then any charged particle projected from outside cannot remain inside forever. Ultimately it has to come out. There may be a number of ways for the charge particle to entre. We discuss a few of them here.

i. If a charged particle enters parallel to Z-axis, it experiences no force hence no acceleration/ retardation hence will go straight and pass out.

ii. If the initial velocity of charge particle is in x-y plane, the particle will experience a force, inside the field region; perpendicular to its velocity as a result it will traverse a circular arc and will come out in any case.

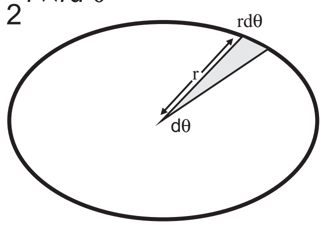
iii. If the initial velocity is perpendicular to a surface, the particle will traverse a semicircle and will come out perpendicular to the surface.

iv. If the initial velocity is not in x-y plane the trajectory will be helical and the particle will ultimately come out.
Over all this is **not possible**

B2. Static magnetic field cannot do work, however, time dependent magnetic fields have associated electric fields and these latter fields do work. The magnetic fields in the vicinity of cranes are non-static and time dependent. Hence it is possible to do work with them. This does not contradict the statement made provided the magnetic field in the statement are required to be to non-static. Hence the statement is defended.

B3 Let the radius vector sweeps an angle $d\theta$ in time dt. Then $\omega(t) = \frac{d\theta}{dt}$ and the area of shaded region is

$dA = \frac{1}{2} r \times r d\theta$ and




$\frac{dA}{dt} = \frac{1}{2} r \times r \frac{d\theta}{dt} = \frac{1}{2} r^2 \times \omega$ is areal velocity.

The angular momentum
$L = I\omega = mr^2\omega$ or $L = m\left(2\frac{dA}{dt}\right) = 2m\frac{dA}{dt}$

Since m is constant, L is conserved when $\frac{dA}{dt} = constant$. Hence statement is true.
Defended.

B4. The problem with this statement is that the colloquial meaning of the word elastic is opposite to its scientific meaning. In the colloquial sense, elastic means easily stretchable, Thus colloquially speaking rubber seems more elastic. Scientifically however, the word elastic has the following meanings:

1. High value of Young's modulus
2. Ability to return to original shape once the forces causing deformation are removed
3. Stress and Strain are related through a single valued function. There is no hysteresis.
4. The departure from single-valuedness viz the onset of plasticity occurs for a very high value of stress.

With respect to all these properties, steel is indeed more elastic than rubber. Hence statement interpreted scientifically stands defended.

B5. The working of a zone plate is based on the theory of Fresnel's half period zones and exhibit focusing action like a convex lens $\frac{1}{a}+\frac{1}{b}=\frac{n\lambda}{r_n^2}$. Where a and b are the conjugate distances of object and image from zone of plate. However it has multiple foci.

The respective focal lengths being

$f_1=\frac{r_n^2}{n\lambda}, f_2=\frac{r_n^2}{3n\lambda}, f_3=\frac{r_n^2}{5n\lambda}$ and soon where

$r_n$ is the radius of os $n^{th}$ half period zone on the zone plate. The statement is defended.

B6. The vibrations of $\vec{E}$ vector of incident light are at $30^0$ with optic axis so the amplitude of the E-ray is A cos $30^0$ and that of O-ray is A sin $30^0$

$$\therefore \frac{\text{The intensity of O-ray}}{\text{The intensity of E-ray}}=\frac{(A\sin 30)^2}{(A\cos 30)^2}=\frac{1}{3}$$

The statement is defended.

B7. Uncertainty principle is expressed

$$\Delta E\times\Delta t=\frac{\hbar}{2}$$

Given that $\Delta t=10^{-8}$sec

$$\therefore\ \Delta E=\frac{6.63\times10^{-34}}{4\pi\times10^{-8}\times1.6\times10^{-19}}=3.29\times10^{-8}\,eV$$

$\Delta E = 3.3\times10^{-8}$ eV and not 3.3eV

Hence refuted

B8. Given that the energy of photon is equal to that of an electron means the electron energy $E=\sqrt{p^2c^2+m_0^2c^4}=h\nu$ further since $m_0c^2$ is negligible, much less than E. Therefore E = pc or $\frac{E}{c}=p$ where as the momentum of photon is $\frac{h\nu}{c}=\frac{E}{c}$ thus the momentum of photon equals that of the electron and so must be the wave length i,e $\lambda_{electron}=\lambda_{photon}$ hence the statement is defended.

B9. Hydrogen nuclei have a greater probability of capturing the colliding neutron through the reaction ${}_1^1H+{}_0^1n\rightarrow{}_1H^2+\zeta\ (energy)$ rather than reducing their energy. The heavy water has negligible cross-section for neutron capture. It only slows down the neutrons. This is why heavy water is a suitable moderator for a nuclear reactor.

B10. Basically without diode, the transistor circuit is a NOT-gate circuit, then the output will be always $\bar{x}$ for input x.

Now diode modified the output. It will allow the current only when diode will be forward bias. If Y input is high then it will be reverse bias and no current will flow through it. Thus output will be high only when both transistor $T_1$ & diode $D_1$ do not conduct otherwise it will be low.

Truth Table

| x | y | z |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

Thus the Boolean expression is as $Z=\bar{x}y$

# Part B2 (Numerical Problems)

P1 (a) $N = N_0 e^{-\lambda t}$ Thereby $\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$

this gives the number of atoms disintegrated in one second. Thus $-\left.\frac{dN}{dt}\right|_{t=1} = \lambda N_0 e^{-\lambda} = \frac{\ln 2}{T}\frac{N_A}{A}\times 1$

$$= \frac{\ln 2}{1620\times 365\times 24\times 3600} \times \frac{6.023\times 10^{23}}{226}\times 1$$

The number of atoms disintegrated per second is $= 3.62 \times 10^{10}$

(b) Given that $-\frac{dm}{dt} \propto m \Rightarrow -\frac{dm}{dt} = k\,m$

$\frac{dm}{m} = -kdt$ Integrating In $m|_{m_0}^{m} = -\,k\,t|_0^t$ or

$m = m_0 e^{-kt}$ or $kt = \ln\frac{m_0}{m}$ where m is the mass left after time t. Now after t = 1 hour, m = (1-0.30) $m_0$ = 0.7$m_0$ and after After time $t$, $m = (1-0.90)m_0 = 0.1\,m_0$ Then

$$t = \frac{\ln 10}{\ln\frac{10}{7}} = \frac{2.30258}{0.3567} = 6.5\,hr \text{ Ans.}$$

P2 $\vec{F}.\vec{dl} = \left(2xy\hat{i} + \left(x^2 - z^2\right)\hat{j} - 3xz^2\hat{k}\right).\left(\hat{i}dx + \hat{j}dy + \hat{k}dz\right)$

$= 2xydx + x^2dy - z^2dy - 3xz^2dz$ Now introducing a new variable t such that

x = 0, y = 0 and z = 0 when t = 0 and

x = 2, y = 1 and z = 3 when t = 1

so that x = 2t ⇒ dx = 2dt y = t ⇒ dy = dt

z = 3t ⇒ dz = 3dt Thereby

$$\int_A^B F.dl = 8\int_0^t t^2dt + \int_0^t \left(4t^2 - 9t^2\right)dt - 162\int_0^t t^3dt$$

$$\int_A^B F.dl = 3\int_0^1 t^2dt - 162\int_0^1 t^3dt = \left.\left(3\frac{t^3}{3} - 162\frac{t^4}{4}\right)\right|_0^1$$

$$\int_A^B Fdl = (1 - 40.5) = -\,39.5\ J$$

or

$\vec{F}.\vec{dx} = \left(2xyi + \left(x^2 - z^2\right)j - 3xz^2k\right).\left(idx + jdy + kdz\right)$

= 2xydx + $x^2$dy - $z^2$dy - 3x$z^2$dz Now

$$\int_A^B \vec{F}.\vec{dl} = 2x\left(\frac{x}{2}\right)dx + \left(2y\right)^2 dy - \left(3y\right)^2 dy - 3\left(\frac{2}{3}z\right) z^2dz$$

$$= \int_0^2 x^2dx + 4\int_0^1 y^2dy - 9\int_0^1 y^2dy - 2\int_0^3 z^3dz$$

$$= \left\{\frac{x^3}{3}\Big|_0^2 - 5\left\{\frac{y^3}{3}\Big|_0^1 - 2\left\{\frac{z^4}{4}\Big|_0^3\right.\right.\right.$$

$$= \frac{1}{3}\times 8 - \frac{5}{3}\times 1 - \frac{2}{4}81 = 1 - \frac{81}{2} = -\,39.5\,J$$

P3. According to Einstein equation, the energy ($hv_1$) of the light-photon incident on the surface is equal to the work-function W of the surface plus the kinetic energy $E_k$ of the emitted photo-electron. Thereby

$hv_1 = W + E_k$

If the retarding potential is $v_1$ then

$E_k = eV_1$ also $v_1 = c / 1\lambda$

$$\therefore \frac{hc}{\lambda_1} = W + eV_1 \quad \Rightarrow \quad W = \frac{hc}{\lambda_1} - eV_1$$

$$= \frac{\left(6.6\times 10^{-34}\right)\times\left(3\times 10^8\right)}{4950\times 10^{-10}} - \left(1.6\times 10^{-19}\right)\times 0.6$$

$$= 4.0\times 10^{-19} - 0.96\times 10^{-19} = 3.04\times 10^{-19}\,J$$

$$or \quad W = \frac{3.04\times 10^{-19}}{1.6\times 10^{-19}} = 1.9\,eV$$

Using the light of another wavelength, ($\lambda_2$), we shall have

$$\frac{hc}{\lambda_2} = W + eV_2 = 3.04\times 10^{-19} + (1.6\times 10^{-19})\times 1.1$$

$= 4.80\times 10^{-19} J$

$$\therefore \lambda_2 = \frac{hc}{4.80\times 10^{-19}} = \frac{\left(6.6\times 10^{-34}\right)\times\left(3\times 10^8\right)}{4.80\times 10^{-19}}$$

$$\lambda_2 = 4125\times 10^{-10}\,m = 4125\,A^0$$

Since the magnetic field does not change the speed of the ejected electrons, there will be no change in the stopping potential in both the cases.

P4 (a) Magnetic induction $\vec{B} = \vec{\nabla}\times\vec{A}$

$$\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & y^2x & -4xyz \end{vmatrix}$$

$$or\ \vec{B} = \hat{i}\left[\frac{\partial}{\partial y}(-\,4xyz) - \frac{\partial}{\partial z}\left(y^2x\right)\right]$$

$$+\hat{j}\left[\frac{\partial}{\partial z}\left(x^2 y\right)-\frac{\partial}{\partial x}(-4xyz)\right]$$

$$+\hat{k}\left[\frac{\partial}{\partial x}\left(y^2 x\right)-\frac{\partial}{\partial y}\left(x^2 y\right)\right]$$

$$\vec{B}=-4xz\hat{i}-0+\hat{j}\times(0+4yz)+\hat{k}\left(y^2-x^2\right)$$

$$\vec{B}=-4xz\hat{i}+4yz\hat{j}+\left(y^2-x^2\right)\hat{k}$$

$$\vec{B}(-1,2,5)=20\hat{i}+40j+3\hat{k}\quad Weber/\mathrm{m}^2$$

b) Magnetic flux

$$\phi=\iint \vec{B}.\vec{ds}$$

Further $\vec{B}.\vec{ds}=\vec{B}.dxdy\,\hat{k}$

$$=\left[-4xz\,\hat{i}+4yz\,\hat{j}+\left(y^2-x^2\right)\hat{k}\right].dx\,dy\,\hat{k}$$

$$\phi=\int_0^1\int_{-1}^4\left(y^2-x^2\right)dx\,dy$$

$$=\int_0^1\int_{-1}^4 y^2dxdy-\int_0^1\int_{-1}^4 x^2dxdy=\left(\frac{xy^3}{3}-\frac{x^3y}{3}\right)\Bigg|_0^1\Bigg|_{-1}^4$$

$$\phi=\frac{(1-0)\left[(4)^3-(-1)^3\right]}{3}-\frac{\left(1^3-0\right)\left[4-(-1)\right]}{3}$$

$$=\frac{60}{3}=20\ Web$$

P5 The maximum frequency is observed when the star is moving directly toward the Earth where as the minimum while moving directly away. For the first case we have

$u=v-V_{CM}$ and the second $u=v+V_{CM}$

Thus

$$f_{max}=4.568910\times10^{14}=4.56811\times10^{14}\left(1-\frac{V_{CM}-v}{c}\right)\text{ and}$$

$$f_{min}=4.567710\times10^{14}=4.56811\times10^{14}\left(1-\frac{v+V_{CM}}{c}\right)$$

Solving for $v$ and $V_{CM}$, we get

$v = 3.9403 \times 10$ (m/s) and

$V_{CM} = -1.3135 \times 10^4$ (m/s)

this means that the binary system is approaching the Earth. The radius R can be found

by $\dfrac{2\pi R}{v}=T, where\ T=11\,days$

$$R=\frac{vT}{2\pi}=5.96\times10^9 m\ =5.96\times10^6 km$$

Now for the circular motion of radius R for any one of the binary stars

$$\frac{Gm^2}{4R^2}=\frac{mv^2}{R}\Rightarrow m=5.546\times10^{29}kg$$

P6. In Michelson's interferometer the interference pattern produced by two close wavelengths $\lambda_1$ and $\lambda_2$ disappears if $\dfrac{2d}{\lambda_2}-\dfrac{2d}{\lambda_1}=\dfrac{1}{2}$ where 2d is the path difference between the two beams producing interference. Thus

$$2d=\frac{\lambda_1\lambda_2}{2(\lambda_1-\lambda_2)}=\frac{\lambda^2}{2(\lambda_1-\lambda_2)}$$ where $\lambda$ is the

mean wavelength of $\lambda_1$ and $\lambda_2$. If instead of two discrete wavelengths, a beam of light having wavelengths lying between $\lambda$ and $(\lambda+\delta\lambda)$ is used then the fringes will disappear if $2d\le\dfrac{\lambda^2}{\delta\lambda}$. This $\delta\lambda$ is known as the line width.

The temporal coherence of the beam, thus is directly associated with the width of the spectral line. Since the fringes are not observed if the path difference exceeds the coherence length L, we may assume that beam contains all the wavelengths lying between $\lambda$ and $(\lambda+\delta\lambda)$.

Then line width $\delta\lambda=\dfrac{\lambda^2}{L}$ further using

$\upsilon=\dfrac{c}{\lambda}$, the frequency spread $\delta\upsilon$ of the line would be $\delta\upsilon=-\dfrac{c}{\lambda^2}\delta\lambda=\dfrac{c}{\lambda^2}\times\dfrac{\lambda^2}{L}=\dfrac{c}{L}$ Further since

conference time $\tau_2=\dfrac{L}{c}\ \therefore\ \delta\upsilon=\dfrac{1}{\tau_2}$ thus the frequency-spread of line is of the order of the reciprocal of coherence time. It is clear that if frequency-spread is known, the coherence-time can be calculated.

Here $\lambda=6058A^0=6.058\times10^{-7}$ m, L = 20 cm

and c = $3\text{x}10^8\ \dfrac{m}{sec}$ Then Line width

$$\delta\lambda=\frac{\lambda^2}{L}=\frac{\left(6.058\times10^{-7}\right)^2}{0.20}$$

$\delta\lambda=0.01835\ 10^{-10}$ m $=0.01835A^0$

Frequency-spread

$$\delta\upsilon=\frac{c}{L}=\frac{3\times10^8}{0.20}=1.5\times10^9 H_Z$$

$$\tau_c=\frac{1}{\delta\upsilon}=\frac{1}{1.5\times10^9}=0.67\times10^{-9}\text{ sec.}$$

P7 Self-inductance: Whenever a current is passed through a coil, a magnetic flux is found to be linked with the coil. The flux so linked is proportional to the current

i,e $\phi \propto i$ or $\phi = Li$ --------- (1)

This phenomenon is known as self-induction and the constant L is known as the coefficient of self-induction or simply the self - inductance of the coil. L is expressed in units of Weber/ampere = Henry. One may also express the induced emf in the coil as $\varepsilon = -\frac{d\phi}{dt} = -L\frac{di}{dt}$

For a current carrying circular coil of radius r magnetic field produced at its centre is

$B = \frac{\mu_0 ni}{2r}$ If r is small, the flux

$\phi = B.A = \frac{\mu_0 ni}{2r} \cdot \pi r^2$

Thereby the flux linked to n turns is

$n\phi = \frac{\mu_0 \pi n^2 r}{2} i = Li$ therefore the Self Inductance $L = \frac{\mu_0 \pi n^2 r}{2}$

Mutual Inductance: Whenever current is passed through a coil, a magnetic flux is found to be linked with the neighbouring coil such that

$\phi_2 \propto i_1$ or $\phi_2 = M_{21} i_1$ ................ (2)

This phenomenon of linkage of flux with the second coil when current is passed through the first coil is known as mutual induction and the constant $M_{21}$ is the coefficient of mutual induction or simply the mutual inductance between the two coils. Unit of M is 'Henry'. Also if the current is passed in second coil, the flux is found to be linked with the first coil i,e $\phi_1 = M_{12} i_2$

It is observed that $M_{12} = M_{12} = M$ and M is known as mutual inductance between the two coils under the given condition.

In case the two coils are in perfect coupling,

$M = \sqrt{L_1 L_2}$ If we consider a long solenoid with n turns per unit length and carrying a current i, the magnetic field at a point on its axis near the middle is $B = \mu_0 ni$ If a coil of small radius rand having N turns is just kept coaxial with the solenoid at its centre, the flux linked with the coil is $\phi_{coil} = NB\pi r^2 = N(\mu_0 ni)\pi r^2$ or

$\phi_{coil} = \mu_0 Nn \pi r^2 i \Rightarrow \phi_{coil} = M i_{solenoid}$

so the mutual inductance between the coil and the solenoid is $M = \mu_0 Nn \pi r^2$ In the present problem a short solenoid is kept coaxial with a long solenoid. If a current I be passed through the long solenoid, flux through the short solenoid is then $\phi_{short} = n_1 l (\mu_0 ni_{long}) \pi a^2$ or

$\phi_{short} = \mu_0 n_1 \ln \pi a^2 i_{long} = M i_{long}$

At the same time $\phi_{long} = Mi_{short}$

$\therefore \phi = \mu_0 n_1 n_2 l \pi a^2 i$ there by the mutualinductance is $M = \mu_0 n_1 n_2 l \pi a^2$

P8 The intensity in the diffraction pattern of double slit is expressed as

$$I = 4R_0^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \cos^2\beta \text{ ....... (1)}$$

Where $\alpha = \frac{\pi}{\lambda} e \sin\theta$ and $\beta = \frac{\pi (e+d) \sin\theta}{\lambda}$

where e and d are the width of each slit and the separation between the two consecutive slits respectively.

Missing order of diffraction: It is seen from equation (1) that the intensity is the product of two terms namely $\left(\frac{\sin\alpha}{\alpha}\right)^2$ which represents the resultant intensity due to single slit and $\cos^2\beta$ which denotes the effect of two slits.

Intensity is maximum when $\cos^2\beta = 1$ showing a maximum. If however the intensity due to single slit vanishes at this position of maximum then this order of maximum is said to be absent hence for absent order

$\sin\alpha = 0$ (Must be) $\Rightarrow e\sin\theta = m\lambda$ and

$\cos^2\beta = 1 \Rightarrow (e+d)\sin\theta = n\lambda$ In the present problem at $\theta\, 5^0$ m = 1, and N = 4

$$\therefore \frac{(e+d)\sin 5}{e \sin 5} = \frac{4}{1} \Rightarrow \frac{e+d}{e} = 4 \Rightarrow d = 3e$$

Also from $e \sin 5^0 = \lambda$ (for first minimum)

$$\Rightarrow e = \frac{440\times10^{-9}}{\sin 5^0} m = \frac{440\times10^{-9}}{0.08716}$$

Thus $e = 5.048\times10^{-6}$ m $= 0.005$ mm and $d = 3\times 5.04810^{-6} = 0.015$mm

P9. According to Ohm's law the resistance of given silicon piece

$$R = \frac{V}{I} = \frac{10}{100\times10^{-3}} = 100\Omega$$

Also $R = \rho\frac{l}{A} = \frac{l}{\sigma A}$

$\therefore$ the conductivity $\sigma = \frac{l}{RA} = \frac{10\times10^{-6}}{100\times0.001\times10^{-4}}\frac{mho}{m}$

or $\sigma = 1.0\frac{mho}{meter}$ Further the current is expressed as $i = neAv_d \times neA\mu_e E$ or

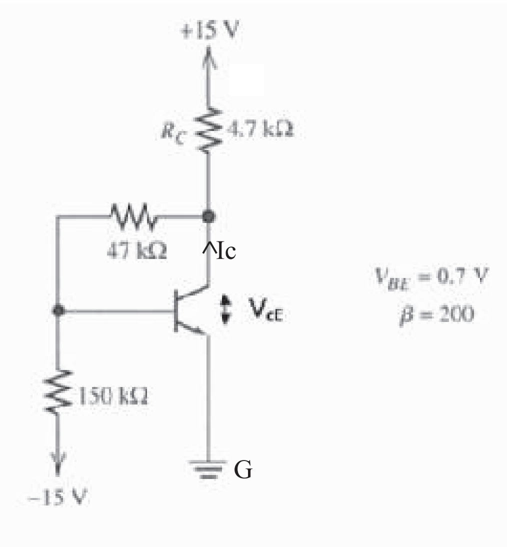
$i = neA\ \mu_e$ or $\frac{V}{l}$

$100\times10^{-3} = n\times1.6 = 10^{-19}\times0.001\times10^{-4}\times0.14$

$0.1 = n\times22.4\times10^{-28} \Rightarrow n = \frac{100}{22.4}\times10^{25}$ per $m^3$

Thus the electron density is $n = 4.45\times10^{25}$ per $m^3$

P10. Appling Kirchhoff's voltage law in the input section of the circuit with $I_B$ as the base current, we get




$$+15V = (I_C + I_B)R_C + I_B 47k\Omega + I_B 150k\Omega - 15V$$

Using now $I_C = \beta I_B$ in above equation

$$30V = [(\beta I_B + I_B)4.7 + I_B\times47 + 150\times I_B]\times1000$$

$$30V = [(200+1)\times4.7 + 197]\times1000\ I_B$$

$$or\ I_B = \left[\frac{30V}{201\times4.7+197}\right]\times10^{-3} = \frac{30V}{1141.7}\times10^{-3}A$$

$$I_B = 26.27\times10^{-6}A = 26.27\mu A$$

Now the collector current $I_c$ is given as -

$$I_C = \beta I_B = 200\times26.27\times10^{-6} = 5.26\ mA$$

Applying Kirchhoff 's voltage law in the output-section

$$15V = I_C R_C + V_{CE}$$

$$V_{CE} = 15 - 5.26mA\times4.7k\Omega$$

$$V_{CE} = 15 - 24.7 = -9.7\ volt$$

Note: The transistor is in saturation mode because both emitter junction and collector junction are in forward bias.